Performance Evaluation 99–100 (2016) 37–54

Contents lists available at ScienceDirect

Performance Evaluation

journal homepage: www.elsevier.com/locate/peva

A unified algorithm for finite and infinite buffer content distribution of Markov fluid models

Hédi Nabli [a,*], Wassim Abbessi [b], Haikel Ouerghi [c]

[a] King Faisal University, College of Science, Department of Mathematics & Statistics, P.O. 380, Ahsaa 31982, Saudi Arabia
[b] University of Manouba, College of Computer Sciences, 9100 Manouba, Tunisia
[c] Jazan University, College of Science and Arts of Eddarb, Department of Mathematics, P.O. Box 114, Jazan, Saudi Arabia

HIGHLIGHTS

- A unified implementing code for all variants of single buffer is proposed.
- The method is based on the uniformization technique that is numerically stable.
- A numerical analysis shows that MAM may be inaccurate: probability exceeding 1.
- Less sensitive in computational time to the number of plotted points than MAM.
- A real application of the theoretical results on a video traffic is achieved.

ARTICLE INFO

Article history:
Received 14 October 2014
Received in revised form 7 January 2016
Accepted 18 February 2016
Available online 2 March 2016

Keywords:
Stochastic fluid model
Markov process
Video traffic

ABSTRACT

This paper is interested in the asymptotic distribution of fluid models with finite or infinite buffer capacity. The probability distribution of the buffer content is controlled by a linear differential system with specific boundary conditions. All earlier methods are forced to test the singularity of the drift matrix before starting any computation. Also, the finite and infinite buffer capacity cases are treated differently. Our approach leads to a unified algorithm which is able to compute simultaneously the buffer workload distribution for finite and infinite buffer capacity without looking into the drift matrix singularity. The proposed method is not spectral and it is based on the uniformization technique. An example of a video traffic is firstly modeled by a Markovian fluid and secondly analyzed.

1. Introduction

The stochastic fluid models are mathematical concepts widely used in telecommunication networks modeling. In reality, the data (cells or packets) sent in the network are actually discrete units. But for heavy traffic, the transmission data is considered as a continuous flow. The fact of assimilating the discrete units transmission as a fluid flow is plausible since the inter-arrival times, for bursty traffic, are negligible compared to the burst times.

For Markovian fluid models, the input and output rates are supposed to be governed by an irreducible Markov process with finite or infinite countable states space. The study of these models shows that the joint distribution of the buffer content and the state of the Markov process satisfies a linear differential system. Many approaches are developed in literature to solve this differential system. Some of them are based on spectral analysis of a key matrix [1–4]. The disadvantage of this approach

* Corresponding author.
E-mail addresses: aalnabli@kfu.edu.sa (H. Nabli), wassim.abbessi@cristal.rnu.tn (W. Abbessi), houerghi@jazanu.edu.sa (H. Ouerghi).

http://dx.doi.org/10.1016/j.peva.2016.02.003

is its sensitivity to eigenvalues computational accuracy [5]. The Laplace transform is also used as a resolution technique especially for transient analysis [6–11]. A numerical inverse transform is then required to derive the joint distribution. The randomization technique is a good alternative to avoid the numerical instability problem [12–15]. Matrix-analytic methods provide efficient algorithms to compute the stationary distribution. These approaches need an algorithm to solve a key matrix equation in order to express the probability distribution by means of exponential matrices [16–22]. Mathematical developments for more general fluid models have been carried out. Some works include a Brownian motion [23–25], others tolerate jumps or phase transition when reaching level zero [26–28] and finally some others are interested either in the fluid models driven by an uncountable-state process [29,30] or in the two-dimensional models [31–34].

The fluid models with Markov modulated input rates have been widely analyzed in several papers. Anick et al. [1] give an explicit expression of the steady-state behavior of buffer content using eigenvalues and eigenvectors of a key matrix. Mitra [3] generalizes this technique by taking the infinitesimal generator as Kronecker products between tridiagonal matrices and identity matrices. Stern and Elwalid [35] deal with inputs generated by a reversible Markov modulated rate process. Sericola and Tuffin [14] used the uniformization technique to derive a new approach for the computation of the stationary buffer content driven by a Markovian queue. Nevertheless, they have assumed that the state space contains only one state with negative effective input rate. In [12,13], Nabli overcame this restriction and considered the general Markov fluid model supporting null effective input rates. In [36], the asymptotic solution is deduced from the transient one, this approach leads to compute numerically the smallest time to reach the stationary regime. Akar and Soharby [37] studied both infinite and finite-buffer cases. A matrix-exponential representation for the buffer content distribution has been proposed. The work given in [38] is an extension of the approaches in [39,40] to the analysis of a Markov modulated fluid flow with a finite buffer. A sequence of finite queues has been constructed in such a way that the work in the system converges to the considered buffer content. Their methods do not use the differential system that governs the joint probability distribution. Bean and O'Reilly [41] derived a spatially-coherent uniformization to approximate the original stochastic fluid model by the discrete level in the Quasi-Birth-and-Death process. Matrix-analytic methods (MAM) have attracted a lot of attention to analyze the standard fluid models. Ramasmawi [19] dealt with the case of infinite buffer capacity and invertible diagonal drift matrix. This work was extended by Soares and Latouche first to singular drift matrix [20] and then to finite buffer capacity [22]. For MAM, it is useful to note that there is no investigation on the impact of the approximation of the key matrix, which is solution of a matrix equation, on the accuracy of the buffer content distribution. In [15], Sericola studied the finite buffer case, where he assumed that the state space contains only one state with negative effective input rate.

In this work, we deal with the general Markov fluid models without any restriction on the nature of the diagonal drift matrix or on the number of negative effective input rates. Also, the probability distribution of the buffer content with infinite capacity will be immediately deduced from the finite case. Moreover, the singular case and the nonsingular case, related to the diagonal matrix **D** composed by all effective input rates, are not discussed separately. A unified algorithm gives simultaneously the probability distribution for finite and infinite buffer capacity without any restrictions on the nature of matrix **D** or on the infinitesimal generator of the Markov process governing the input and output rates. At the best of our knowledge, this is the first paper that deals with all these variants by a single implementing code. Furthermore, our method is based on randomization which is acknowledged by its numerical stability and its accuracy. It is a natural extension of [12,14,15]. Finally, our algorithm will be applied on a real video traffic which is beforehand modeled by a Markovian stochastic fluid model. It should be noted that our main result uses a conjecture which is similar to the one made in [36,42]. This conjecture will be proved for the case where the state space allows only one negative effective input rate.

The rest of the paper is organized as follows. In Section 2, we describe the mathematical model. The finite buffer case is considered in Section 3 where we obtain the distribution of the stationary buffer content. The infinite buffer case will be easily deduced from the finite buffer case. These two results are independent from the nature of matrix **D**, no matter if it is singular or invertible. Section 4 is devoted to explain our methodology in computing the probability distribution of the buffer content for both finite and infinite buffer capacity. In Section 5, we present numerical illustrations of these results. More precisely, a video traffic is modeled by a general Markov fluid process. Our mathematical approach is then applied on this model and the numerical results are illustrated and analyzed. The benefits in terms of accuracy and time complexity will be quantitatively substantiated. Finally, Section 6 summarizes our contribution.

2. Model description and notation

We consider a fluid model with a buffer of capacity B, for which the input and output rates are controlled by an irreducible Markov process $(X_t)_{t\geqslant 0}$ over a finite state space S with infinitesimal generator $\mathbf{A} = (a_{ij})_{i,j\in S}$. The input (*resp.* output) rate at state i is denoted by r_i (*resp.* c_i). While the process $(X_t)_{t\geqslant 0}$ stays in state i, the fluid arrives to the buffer at rate $r_i \geqslant 0$ and the server empties the buffer at rate $c_i \geqslant 0$. When the buffer content grows to the capacity B, the incoming fluid is wasted. Let $d_i = r_i - c_i$ be the effective input rate at state i. Also, let $\boldsymbol{\pi} = (\pi_i, i \in S)$ be the steady-state probability vector of $(X_t)_{t\geqslant 0}$. Since $\mathbf{A}$ is irreducible, it is well-known that $\boldsymbol{\pi}$ is the unique solution of the linear system $\boldsymbol{\pi}\mathbf{A} = \mathbf{0}$ and $\sum_{i\in S} \pi_i = 1$, where $\mathbf{0}$ is the row vector of dimension the cardinality of S with all entries equal to 0.

Let Q_t^B (*resp.* Q_t) be the quantity of fluid in the buffer at time t when $B < \infty$ (*resp.* $B = \infty$). Let Q^B (*resp.* Q) be the limit of Q_t^B (*resp.* Q_t), and $F_i^B(x) = \mathbb{P}\{Q^B \leqslant x, X_\infty = i\}$ (*resp.* $F_i(x) = \mathbb{P}\{Q \leqslant x, X_\infty = i\}$) be the limit of $\mathbb{P}\{Q_t^B \leqslant x, X_t = i\}$ (*resp.* $\mathbb{P}\{Q_t \leqslant x, X_t = i\}$) when t tends to infinity. Let us denote by $\mathbf{D}$ the diagonal matrix composed by the effective input rates: