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Reconstructing graphs from size and degree properties of their induced k-subgraphs

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Abstract

As a variant of the famous graph reconstruction problem we characterize classes of graphs of order n such that all their induced subgraphs on $k \leqslant n$ vertices satisfy some property related to the number of edges or to the vertex degrees.

We give complete solutions for the properties (i) to be regular, (ii) to be regular modulo $m \geqslant 2$ or (iii) to have one of two possible numbers of edges. Furthermore, for an order n large enough, we give solutions for the properties (iv) to be bi-regular or (v) to have a bounded difference between the maximum and the minimum degree.
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1. Introduction

In this paper we consider the following variant of the famous *reconstruction problem* [3,9,11]: *Given the information that all induced subgraphs on k vertices of an unknown graph G on $n \geqslant k$ vertices satisfy a certain property*, *can we determine the graph G*?

While in the classical reconstruction problem we are given all induced k-subgraphs of G—the so-called *k-deck* of G—and try to reconstruct G, here we are only given some information on a constrained k-deck and we would like to know if this information suffices to reconstruct the graph or the family of graphs whose k-deck satisfies the constraints.

This kind of problem has been considered under different names and in various variants. The first paper in this spirit was written in 1960 by Kelly and Merriell [10]. More recent papers considered the problem to characterize graphs having the following properties:

(P_1) All induced k-subgraphs have the same number of edges [4,8] (cf. also [12, p. 50]).
(P_2) All induced k-subgraphs have the same number of edges modulo m [7].
(P_3) All induced k-subgraphs have the same domination number [5].
(P_4) For all induced k-subgraphs some complete graph parameter assumes the same value. A parameter π is called complete, if for every $k \geqslant 2$ there are real numbers $a_k \leqslant b_k$ such that $\{H=(V_H,E_H) \mid |V_H|=k, \pi(H) \in \{a_k,b_k\}\}=\{K_k,\bar{K}_k\}$ [4,5].

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Related algorithmic aspects and complexity results were obtained in [6,7] and in [1] realizability and uniqueness problems were addressed.

In the present paper we extend this line of research by characterizing graphs of order n having one of the following five properties:

(P_5) All induced k-subgraphs are regular (not necessarily of the same degree).
(P_6) All induced k-subgraphs are regular modulo m (not necessarily of the same residual degree).
(P_7) All induced k-subgraphs have one of two possible numbers of edges (generalizing P_1).
(P_8) All induced k-subgraphs are bi-regular (generalizing P_5).
(P_9) All induced k-subgraphs have a bounded difference between the maximum and the minimum degree (again generalizing P_5).

For properties P_5, P_6 and P_7 we give complete solutions for all possible values of k and n. We also provide new shorter proofs for the known results about properties P_1 and P_2. For properties P_8 and P_9 we give complete solutions for every k and sufficiently large n.

Note that the properties P_1, P_2 and P_5–P_9 are symmetric in the sense that some graph has the property if and only if its complement has it. We will use this symmetry often during our proofs.

Our paper is organized as follows. In Section 2 we present new shorter proofs for two known results and solve problems P_5 and P_6. In Section 3 we solve problems P_7 and P_8 and in Section 4 we solve problem P_9. Finally, in Section 5 we pose some problems.

Before we proceed to Section 2 we introduce some notation (for notation not defined here please refer to [2,12]). We consider finite, undirected and simple graphs $G=(V,E)$ with vertex set V, edge set E, order $n=|V|$ and size $|E|$. The neighbourhood and degree of a vertex $u\in V$ in the graph G are denoted by $N_G(u)$ and $d_G(u)$, respectively. The maximum and minimum degree of G are denoted by $\Delta(G)$ and $\delta(G)$, respectively. The graph G is regular if $\Delta(G)=\delta(G)$ and *biregular* if G has at most two different vertex degrees, i.e. $|\{d_G(u) \mid u\in V\}|\leqslant 2$. If $m\geqslant 2$ is an integer, then G is *regular modulo m* if $d_G(u)\equiv d_G(v) \pmod m$ for all $u,v\in V$.

For $X\subseteq V$ let $G[X]$ denote the subgraph of G induced by X and if $|X|=k$, then $G[X]$ is called an induced k-subgraph of G. The complement of G is denoted by $\bar{G}$.

A set of pairwise adjacent (non-adjacent) vertices is complete (independent). A complete subgraph of G is a clique. For non-negative integers r and s let $R(r,s)$ denote the classical Ramsey number, i.e. every graph of order at least $R(r,s)$ contains either a complete set of cardinality r or an independent set of cardinality s.

Let $G=(V_G,E_G)$ and $H=(V_H,E_H)$ be two graphs with $V_G\cap V_H=\emptyset$. The join $G+H$ of G and H has vertex set $V_G\cup V_H$ and edge set $E_G\cup E_H\cup\{uv \mid u\in V_G, v\in V_H\}$. The union $G\cup H$ of G and H has vertex set $V_G\cup V_H$ and edge set $E_G\cup E_H$.

The complete graph of order n is denoted by K_n. The complete multipartite graph with partite sets of cardinalities $n_1,n_2,\ldots,n_p$ is denoted by $K_{n_1,n_2,\ldots,n_p}$. The path and cycle of order n are denoted by P_n and C_n, respectively.

2. One allowed size or degree

We begin with a new proof for the result of [7] concerning P_2.

Theorem 2.1 (*Caro and Yuster [7]*). *Let $k,m\geqslant 2$ be integers. Let $G=(V,E)$ be a graph of order $n\geqslant k+1$. All induced k-subgraphs of G have the same number of edges modulo m if and only if*

(i) *either $n=k+1$ and G is regular modulo m,*
(ii) *or $n\geqslant k+2$ and G or $\bar{G}$ is*
  (a) *either K_n,*
  (b) *or $K_{1,n-1}$ and $k\equiv 1 \pmod m$,*
  (c) *or $K_{a,n-a}$, $m=2$ and $k\equiv 1 \pmod 2$.*

Proof. Since the 'if'-part is obvious, we prove the 'only if'-part.

Let $G=(V,E)$ be a graph of order $n\geqslant k+1$ and let all induced k-subgraphs of G have the same number of edges modulo m. Obviously, if $n=k+1$, then every vertex must have the same degree modulo m and we are done. Hence,

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