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Analysis of degrees of freedom under mixture Gaussian model in cognitive radio systems

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ABSTRACT

We propose a mixture-Gaussian model for a cognitive radio channel to analyze the interplay between the interference in the system and the degrees-of-freedom (DOF), i.e., the average number of channel uses per transmission frame, used by the secondary user (SU) for communications in the long run. In contrast to the conventional studies, we assume that the SU receiver (SU-RX) does not precisely know whether the primary-user (PU) transmitter is on or off. Due to this assumption the resulting interference channel is mixture-Gaussian. Our objective is to find the optimal sensing threshold and sensing time for the signal detector used by the SU transmitter (SU-TX). Our formulation of the optimization problem reflects the trade-off between SU-TX's DOF for communications and that for detection. Both the DOFs affect PU's interference to SU, and SU's interference to PU. The latter interference causes PU performance degradation, which is kept within tolerable range as a constraint. As a further contribution, we define interference regimes for SU performance on the basis of PU transmission power level. We also address the scenario when PU receiver uses the nearest neighbor decoding while wrongly anticipating that the channel is Gaussian. Finally, we demonstrate that even if SU-TX's signal detector performs suboptimally, SU can still achieve the optimal detector's performance in the high interference regime by adjusting the sensing parameters.

1. Introduction

The users of sophisticated broadband wireless services are increasing day by day, thereby raising the demand of these services. Therefore, already established networks are expanding their resources and the new service providers are establishing their infrastructure. This gives rise to the problem of scarce bandwidth resources. Since most of the available spectrum has already been licensed, there is almost no room left for accommodating the new demands. There are studies like [1] which show that vast regions of licensed spectrum are underutilized. These are called *white-spaces*. A prospective solution to the problem of scarce bandwidth resources is to use these white spaces for the new wireless systems instead of issuing the new licenses. The device that can help us achieve this goal of utilizing the unused channel is the *cognitive radio* [2,3]. It is a radio that can sense and learn, as the word '*cognitive*' indicates. It has intelligent capabilities to sense the communication activities over the channel and looks for the opportunities available for itself.

In a cognitive radio (CR) based system, there are two categories of users. One of them is called the *primary user* (PU) and the other is called the *secondary user* (SU). PU's are the users which possess the license for using the channel and can use it for communications at anytime. On the other hand, SU's are the unlicensed users and can only use the channel for communications when it is idle. These SU's form the nodes of the cognitive radio network and are

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intelligent enough to sense whether the channel is being used by a PU or not. This is referred to as the *channel sensing* ability of the SU. Also, SU should leave an occupied channel whenever a PU starts transmission using that channel.

We consider a simple cognitive radio system consisting of two transmitter–receiver pairs, one for the primary user PU and the other for the secondary user SU. They have a common frequency channel to use for communications and SU communicates whenever the channel is sensed idle. In order to find out whether the channel is occupied by a PU signal or not, SU transmitter uses a signal detector, e.g., the energy detector. For this system, the main design problem is to find the optimal sensing time and threshold for the detector used by SU transmitter. Here we would like to mention an important remark, i.e., for the rest of the discussion, the *signal detector of SU* refers to the *signal detector used by the SU transmitter*. The optimality criterion is to maximize the SU performance while keeping the detection probability above an appropriate threshold level. This criterion is also called the *sensing-throughput trade-off* for SU.

Note that in this study, *coherence* means the availability of the channel state information (CSI) to the receiver [4,5]. In our model, CSI for the SU receiver is the precise knowledge of the PU transmitter's transmission state, i.e., whether it is on or off. The sensing-throughput trade-off and analysis models have been explored in [6–19], under an assumption that SU receiver operates coherently with both SU and PU transmitters, i.e., SU receiver precisely knows if SU and PU transmitters are on or off. We call this the *conventional model*. However, in this paper, we consider a more practical scenario where SU receiver operates coherently with SU transmitter but *incoherently* with PU transmitter, as in our previous work [20]. This means that SU receiver precisely knows if SU transmitter is on or off, but it does not know the same about PU transmitter. This model appropriately incorporates the interference experienced by both PU and SU systems and is practically more rigorous than the conventional model. To the best of our knowledge, despite such significance, this model has never been studied.

The assumption that whether the SU receiver operates coherently or incoherently with the PU transmitter, determines the nature of the interference channel in the system model. In the conventional model, due to the coherent operation of SU receiver and PU transmitter, the underlying channel is a regular Gaussian interference channel. On the other hand, in our model, due to the incoherent operation of SU receiver and PU transmitter, the resulting interference channel is mixture-Gaussian, as explained in Section 2. Due to the mixture-Gaussian nature of the interference channel, the optimal sensing time and threshold design problem can be better elaborated with the help of a novel concept of a trade-off, that we propose formally in Section 5.1 as the trade-off between the degrees of freedom available to SU for communications and the interference experienced by SU due to PU, rather than that of the conventional sensing-throughput trade-off. In this study, the degrees of freedom available to SU for communications quantify the average number of channel-uses per transmission frame, which is used by SU for communications in the long run, i.e., for infinitely large number of transmission frames, as explained in detail in Section 5.1. We properly define and elaborate the novel concept of the trade-off in Section 5.1, and before proceeding to it, we also build the required background in the subsequent sections, in order to help the reader understand the concept nicely.

Due to the incoherence assumption between SU receiver and PU transmitter, the analysis becomes more challenging. The first challenge is that the interference channel becomes mixture Gaussian that requires the evaluation of the entropy of mixture Gaussian random variables, which does not have a closed form solution. The second challenge is the non-linear interdependence of quite a few parameters. Thus, the optimization problem under consideration becomes quite complicated and cannot be solved using the conventional optimization techniques, as explained in Section 3.2. Also, it does not have a closed form solution, and therefore, we develop numerical algorithms to solve it.

Another loose end in the conventional studies is that an arbitrarily high detection-probability threshold is selected for the detection-probability constraint. In order to tie up this loose end, we select a meaningful value for this threshold which ensures that the PU performance degradation remains within a tolerable range. This requires us to explicitly compute PU performance. While evaluating PU performance, we assume that PU receiver does not know the state of SU transmitter, i.e., PU receiver also operates incoherently with SU transmitter, in the same way as we assume that SU receiver operates incoherently with PU transmitter. Note that a degradation in PU performance is caused by the interference due to SU, when the detection probability is less than 1.

In [20] the sensing-throughput trade-off was explored for a binary coded communication system, the optimization problem was formulated only and some special cases about sensing and throughput trade-off were investigated. In this paper, we extend our previous work in [20] by solving the optimization problem for a more general coded case. As a further novel contribution, we identify a performance inefficiency region for SU at a very high detection probability, where SU performance drastically decreases with increase in detection probability. We call this the *energy detector's inefficiency region* and is abbreviated as EDI region. We also identify the interference regimes for SU performance on the basis of PU transmission power level $\mathcal{P}_{\mathrm{PU}}$. Our study reveals that the tolerance level of PU performance can be exploited to increase the SU performance by sacrificing the detection probability, but only for the *low interference regime for SU*. In this regime, the PU transmission power is weak and at a level lower than some threshold, such that the SU performance increases with decrease in detection probability. On the other hand, for the high interference regime in which the PU power is stronger, the SU cannot sacrifice the detection probability at all. This is because the interference caused by PU is so high that the SU performance decreases with decrease in detection probability for the high interference regime. Therefore, during this regime, the detection probability needs to be kept at the maximum level for the non-EDI region, such that it is greater than the required minimum threshold but small