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Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 53 (2016) 15–24 www.elsevier.com/locate/endm

Intersection graphs of cyclic subgroups of groups

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Abstract

Let G be a group. *The intersection graph of cyclic subgroups of* G, denoted by $\mathscr{I}_c(G)$, is a graph having all the proper cyclic subgroups of G as its vertices and two distinct vertices in $\mathscr{I}_c(G)$ are adjacent if and only if their intersection is non-trivial. In this paper, we classify the finite groups whose intersection graphs of cyclic subgroups are one of totally disconnected, complete, star, path, cycle. We show that for a given finite group G, $girth(\mathscr{I}_c(G)) \in \{3, \infty\}$. Moreover, we classify all finite non-cyclic abelian groups whose intersection graphs of cyclic subgroups are planar. Also for any group G, we determine the independence number, clique cover number of $\mathscr{I}_c(G)$ and show that $\mathscr{I}_c(G)$ is weakly α-perfect. Among the other results, we determine the values of n for which $\mathscr{I}_c(\mathbb{Z}_n)$ is regular and estimate its domination number.

Keywords: Intersection graph, cyclic subgroups, girth, weakly α-perfect, planar.

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http://dx.doi.org/10.1016/j.endm.2016.05.003
1571-0653/

1 Introduction

Associating graphs to algebraic structures and studying their properties using the methods of graph theory has been an interesting topic for mathematicians in the last decade. For a given family $\mathcal{F} = \{S_i \mid i \in I\}$ of sets, the intersection graph of $\mathcal{F}$ is a graph with the members of $\mathcal{F}$ as its vertices and two vertices S_i and S_j are adjacent if and only if $i \neq j$ and $S_i \cap S_j \neq \emptyset$. For the properties of these graphs and several special class of intersection graphs, we refer the reader to [7]. In the past fifty years, it has been a growing interest among mathematicians, when the members of $\mathcal{F}$ have some specific algebraic structures. In 1964 Bosak [2] defined the intersection graphs of semigroups. Motivated by this, Csákány and Pollák [4] defined the intersection graph of subgroups of a finite group.

Let G be a group. *The intersection graph of subgroups of* G, denoted by $\mathscr{I}(G)$, is a graph having all the proper subgroups of G as its vertices and two distinct vertices in $\mathscr{I}(G)$ are adjacent if and only if the intersection of the corresponding subgroups is non-trivial.

Some properties of the intersection graphs of subgroups of finite abelian groups were studied by Zelinka in [13]. Inspired by these, there are several papers appeared in the literature which have studied the intersecting graphs on algebraic structures, viz., rings and modules. See, for instance [1,3,9,10,12] and the references therein.

In this paper, for a given group G, we define *the intersection graph of cyclic subgroups of* G, denoted by $\mathscr{I}_c(G)$, is a graph having all the proper cyclic subgroups of G as its vertices and two distinct vertices in $\mathscr{I}_c(G)$ are adjacent if and only if their intersection is non-trivial. Clearly $\mathscr{I}_c(G)$ is a subgraph of $\mathscr{I}(G)$ induced by all the proper cyclic subgroups of G. When G is cyclic, then $\mathscr{I}_c(G)$ and $\mathscr{I}(G)$ are the same. In [3], Chakrabarty *et al* studied several properties of intersection graphs of subgroups of cyclic groups.

Now we recall some basic definitions and notations of graph theory. We use the standard terminology of graphs (e.g., see [5]). Let G be a graph. We denote the degree of a vertex v in G by $deg(v)$. A graph whose edge set is empty is called a *null* graph or *totally disconnected* graph. K_n denotes the complete graph on n vertices. $K_{m,n}$ denotes the complete bipartite graph with one partition consists of m vertices and other partition consists of n vertices. In particular $K_{1,n}$ is called a *star*. P_n and C_n respectively denotes the path and cycle with n edges. A graph is *regular* if all the vertices have the same degree. A graph is *planar* if it can be drawn in a plane such that no two edges intersect except (possibly) at their end vertices. The girth of