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# Gregarious Kite Decomposition of Tensor Product of Complete Graphs

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**Abstract**

A kite decomposition of a multipartite graph is said to be gregarious if every kite in the decomposition has all its vertices in different partite sets. In this paper, we show that there exists a gregarious kite decomposition of $K_m \times K_n$ if and only if $mn(m-1)(n-1) \equiv 0 \pmod 8$, where $\times$ denotes the tensor product of graphs.

*Keywords:* Kite, Tensor product, Gregarious kite decomposition.

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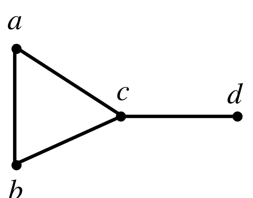
# 1 Introduction

A *latin square* of order $n$ is an $n \times n$ array, such that each row and each column of the array contains each of the symbols from $\{1, 2, \ldots, n\}$ exactly once. A *quasigroup* of order $n$ is a pair $(Q, \circ)$, where $Q$ is a set of size $n$ and "$\circ$" is a binary operation on $Q$ such that for every pair of elements $a, b \in Q$, the equations $a \circ x = b$ and $y \circ a = b$ have unique solutions. It is well known that the multiplication table of a quasigroup defines a latin square. The terms latin square and quasigroup are interchangeable. A quasigroup $(Q, \circ)$ is called *idempotent* if the identity $x^2 = x$ holds for all $x$ in $Q$. A latin square of order $n$ is said to be *idempotent* if cell $(i, i)$ contains symbol $i$ for $1 \leq i \leq n$.

Partition of $G$ into subgraphs $G_1, G_2, \ldots, G_r$ such that $E(G_i) \cap E(G_j) = \emptyset$ for $i \neq j \in \{1, 2, \ldots, r\}$ and $E(G) = \cup_{i=1}^{r} E(G_i)$ is called *decomposition* of $G$; In this case we write $G$ as $G = G_1 \oplus G_2 \oplus \ldots \oplus G_r$, where $\oplus$ denotes edge-disjoint sum of subgraphs. For an integer $\lambda$, $\lambda G$ denotes $\lambda$ copies of $G$.

The *tensor product* $G \times H$ and the *wreath product* $G \otimes H$ of two graphs $G$ and $H$ are defined as follows: $V(G \times H) = V(G \otimes H) = \{(u, v) \mid u \in V(G), v \in V(H)\}$. $E(G \times H) = \{(u, v)(x, y) \mid ux \in E(G)$ and $vy \in E(H)\}$ and $E(G \otimes H) = \{(u, v)(x, y) \mid u = x$ and $vy \in E(H)$ or $ux \in E(G)\}$. It is well known that tensor product is commutative and distributive over an edge-disjoint union of subgraphs, that is, if $G = G_1 \oplus G_2 \oplus \ldots \oplus G_r$, then $G \times H = (G_1 \times H) \oplus (G_2 \times H) \oplus \ldots \oplus (G_r \times H)$. A graph $G$ having partite sets $V_1, V_2, \ldots, V_m$ with $\mid V_i \mid = n$, $1 \leq i \leq n$ and $E(G) = \{uv \mid u \in V_i$ and $v \in V_j, \forall\, i \neq j\}$ is called *complete m-partite graph* and is denoted by $K_m(n)$. Note that $K_m(n)$ is same as the $K_m \otimes I_n$, where $I_n$ is an independent set on $n$ vertices.

A *kite* is a graph which is obtained by attaching an edge to a vertex of the triangle, see $Fig.1$. We denote the kite with edge set $\{ab, bc, ca, cd\}$ by $(a, b, c; cd)$.




$Fig.1.$

A subgraph of a multipartite graph $G$ is said to be *gregarious* if each of its vertices lies in different partite sets of $G$.