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Generating Graceful Trees from Caterpillars by Recursive Attachment

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Abstract

A *graceful labeling* of a graph G with n edges is an injection $f : V(G) \to \{0, 1, 2, \cdots, n\}$ with the property that the resulting edge labels are also distinct, where an edge incident with vertices u and v is assigned the label $|f(u) - f(v)|$. A graph which admits a graceful labeling is called a *graceful graph*. In this paper, inspired by Koh [9] method, which combines a known graceful trees to obtain a larger graceful trees, we introduced a new method of combining graceful trees called recursive attachment method, and we show that the recursively attached tree $T_i = T_{i-1} \oplus T^{A_{i-1}}$ is graceful, for $i \geq 1$, where T_0 is a base tree which is taken as a caterpillar and $T^{A_{i-1}}$ is an attachment tree which taken as any caterpillar. Here $T_{i-1} \oplus T^{A_{i-1}}$ represents a tree obtained by attaching a copy of $T^{A_{i-1}}$ at each vertex of degree at least two in T_{i-1}, for $i \geq 1$. Consequently the graceful tree conjecture is true for every recursively attached caterpillar tree T_i, for $i \geq 1$.

Keywords: Graceful Tree;Graceful Tree Conjecture;Graph Labeling

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1 Introduction

A graph labeling is an assignment of numbers to the vertices or edges, or both of the graph, subject to certain conditions, that are posed from the application problem or theoretical problem. At the Smolenice Symposium in 1963, Ringel posed his celebrated conjecture that the complete graph, K_{2n+1} can be decomposed into $2n+1$ copies of any tree with n edges. In 1965, Kotzig [10] also conjectured a special version of Ringel Conjecture that the complete graph K_{2n+1} can be cyclically decomposed into $2n+1$ copies of any tree with n edges. In an attempt to settle these two conjectures Rosa [11] introduced an hierarchial series of 'valuations' called ρ, σ, β and α-valuations of a graph and used these valuations as a tool to investigate the cyclic decomposition of complete graphs. Later, Golomb [5] called the β-valuation as "graceful labeling" and now this term is most widely used. A *graceful labeling* of a graph G with n edges is an injection $f : V(G) \to \{0, 1, 2, \cdots, n\}$ with the property that the resulting edge labels are also distinct, where an edge incident with vertices u and v is assigned the label $|f(u) - f(v)|$. A graph which admits a graceful labeling is called a *graceful graph*. Further Rosa proved in his classical paper that, if a tree T with n edges has graceful labelling then the complete graph K_{2n+1} can be cyclically decomposed into $2n+1$ copies of a tree. This result leads to the birth of the celebrated Ringel-Kotzig-Rosa Conjecture, which states that every tree is graceful. This conjecture is also popularly called Graceful Tree Conjecture. Graceful Tree Conjecture is shown to be true for any tree with at most 29 vertices by Michal Horton [6]. Some special classes of trees like caterpillar [11], all the diameter five trees [7], banana trees [8], etc.(refer [4]), are all shown to be graceful.

Stanton and Zarnke [12] and Koh and et al.[9] have initiated the construction of combining known graceful trees to obtain a larger graceful trees. In this direction a few interesting results were also proved (refer [4]). Inspired by the Koh's method, we construct graceful trees T_i recursively from caterpillars. That is, the tree $T_i = T_{i-1} \oplus T^A$, for $i \geq 1$ is constructed from T_{i-1} by attaching any fixed internal vertex of each copy of T^A to each internal vertex of T_{i-1}. Here the tree T_0 and T^A are caterpillars. The interesting part of these trees is that they are not only graceful but they also admit the stronger version of graceful labelling called α - labeling. Consequently, due to the results of Rosa [11] and El-Zanati et al. [3] the above recursively constructed tree T_i decomposes complete graph K_{2cm+1} and complete bipartite graph $K_{mp,mq}$ for any positive integer p and q where m is the number of edges of T_i.