



Further Results on Graceful Directed Graphs

S.M.Hegde ¹

*Department of Mathematics and Computational Sciences
National Institute of Technology Karnataka, surathkal
Srinivasnagar-575025 Mangalore, India*

Kumudakshi ²

*Department of Mathematics
NMAM Institute of Technology
Nitte-574110 Udupi Dist.Karnataka , India*

Abstract

In this paper we present the gracefulness of the directed graph $\overrightarrow{P_m \square P_n}$ which is an orientation of the planar grid graph $P_m \square P_n$, in which each cell is a unicycle of length four.

Keywords: Graceful digraph, Grid graph.

1 Introduction

For standard notations and terminologies in graph theory we follow Chartrand and Lesniak [4].

¹ Email: smhegde@nitk.ac.in

² Email: s.kumudakshi@gmail.com

A labeling of a graph is an assignment of real values or subsets of a set to the vertices and edges or both with respect to certain conditions. Labeling of graph plays an important role in the application of graph theory such as coding theory X-ray crystallography, radar, astronomy, circuit design etc. According to Beineke and S.M. Hegde [2] graph labeling serves as a frontier between number theory and the structure of graphs. For more detailed study of graph labeling problems one can refer to Gallian survey [5].

The concept of graceful labeling was introduced by Rosa [7] in the year 1967.

Definition 1.1 An undirected graph with e edges is gracefully labeled if each vertex v is assigned a distinct value $f(v)$ from $\{0, 1, \dots, e\}$ in such a way that the set of edge labels equals $\{1, 2, \dots, e\}$ when edge uv is labeled by $f(u, v) = |f(u) - f(v)|$. A graph is said to be graceful graph if it can be gracefully labeled.

The concept of graceful labeling of undirected graphs was extended to digraphs by Bloom and Hsu in [3].

Definition 1.2 A digraph D with p vertices and q arcs is labeled by assigning a distinct integer value $g(v)$ from $\{0, 1, 2, \dots, q\}$ to each vertex v . The vertex values, in turn, induce a value $g(u, v)$ on each arc (u, v) where $g(u, v) = (g(v) - g(u)) \pmod{(q+1)}$. If the arc values are all distinct, then the labeling is called a graceful labeling of a digraph.

Use of modular arithmetic ties graceful digraph to a variety of algebraic problems of long standing as discussed in [3].

Bloom and Hsu in their introductory paper [3] have observed that a graceful graph always gives rise to a graceful digraph, start with any gracefully labeled undirected graph G with vertex labeling $f(u)$ for vertex u . Simply orienting the edges of G to point towards the larger vertex value produces a graceful digraph D with G as its underlying graph. Thus, if $f(u) > f(v)$, then the edge uv is labeled as $f(u, v) = |f(u) - f(v)|$ which results in the same value being assigned to the corresponding edges in G and D .

Acharya and Gill [1] have investigated graceful labeling for the grid graph $P_m \square P_n$. (A grid is a graph defined as the cartesian product of two graphs).

Note 1 Since the graph $P_m \square P_n$ is graceful one can see that there exists at least one orientation of $P_m \times P_n$ which is graceful.

In this paper we prove the gracefulness of the digraph $\overrightarrow{P_m \square P_n}$ which is a certain non-trivial orientation of the planar grid graph $P_m \square P_n$.

Download English Version:

<https://daneshyari.com/en/article/4651537>

Download Persian Version:

<https://daneshyari.com/article/4651537>

[Daneshyari.com](https://daneshyari.com)