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Dichromatic Polynomial of Product Digraphs

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Abstract

A k-colouring of a digraph $D = (V, A)$ is a partition of $V(D)$ into k acyclic sets. The minimum integer k for which there exists a k-colouring of D is the *dichromatic number* $\chi_d(D)$ of the digraph D. In this paper, we obtain dichromatic polynomial of product digraphs.

Keywords: Colouring of a Digraph, Dichromatic Number, Tensor Product, Cartesian Product, Dichromatic Polynomial.

1 Introduction

A *k-colouring* of a digraph $D = (V, A)$ is a partition of $V(D)$ into k acyclic sets. This colouring parameter was first introduced by Neumann Lara [5]. The minimum integer k for which there exists a k-colouring of D is the *dichromatic number* $\chi_d(D)$ of the digraph D. For more ideas in this colouring, please refer

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[2]. Let D_1=(V_1, A_1) and D_2=(V_2, A_2) be two digraphs. $u \to v$ means there is an arc from u to v. The *tensor product* of the digraphs D_1 and D_2, denoted by $D_1 \times D_2$, has the vertex set $V(D_1 \times D_2) = V_1 \times V_2$ and $(u_1, v_1) \to (u_2, v_2)$ in $D_1 \times D_2$ if and only if, $u_1 \to u_2$ in D_1 and $v_1 \to v_2$ in D_2. The *Cartesian product* of the digraphs D_1 and D_2, denoted by $D_1 \Box D_2$, has the vertex set $V(D_1 \Box D_2) = V_1 \times V_2$ and $(u_1, v_1) \to (u_2, v_2)$ in $D_1 \Box D_2$ if and only if, either $u_1 = u_2$ in D_1 and $v_1 \to v_2$ in D_2 or $u_1 \to u_2$ in D_1 and $v_1 = v_2$ in D_2. For a given digraph D and a given set of λ colours, the function $P(D; \lambda)$ is defined to be the number of ways of colouring D using the λ colours. The digraph $D.a$ is obtained from contracting the arc a, where $a = uv$ be any arc of D. For other notations in graphs and digraphs, please refer [1,3].

A *directed walk* in D is a finite alternating sequence $v_0a_1v_1a_2v_2...a_kv_k$ in which a_i is an arc joining v_{i-1} with v_i, for $i = 1, 2, 3, ..., k$. A *directed path* is a directed walk $v_0a_1v_1a_2v_2...a_kv_k$, in which all $v_i^{'}s$ are distinct. A directed walk $v_0a_1v_1a_2v_2...a_kv_k$ is a *directed cycle* if all $v_i^{'}s$ are distinct except v_0 and v_k and $v_0 = v_k$. For a digraph D, the *underlying graph* of D, denoted by $G(D)$, has the same vertex set as that of D, where two vertices u and v are adjacent if uv or vu is an arc in D.

2 Main Results

We begin this section with some lemmas.

Lemma 2.1 [2] *D is an acyclic digraph if, and only if, $\chi_d(D) = 1$.*

Lemma 2.2 [2] *Let D be a digraph without digons. Then $\chi_d(D) = 2$ if, and only if, D has a directed cycle.*

Observation 2.3 *Let us take two directed cycle of length l and m and $l \leq m$. If we rotate the directed cycle of length l and m, then we get a directed cycle of length r where $r = [l, m]$. If we take all the meeting points of the two directed cycles as vertices and rotations as arcs, then we get a directed cycle of length r.*

Observation 2.4 *If l and m are odd, then $r = [l, m]$ is odd. If l and m are even, then $r = [l, m]$ is even. If l is even and m is odd, then $r = [l, m]$ is even.*

Lemma 2.5 *$D_1 \times D_2$ has a directed cycle if, and only if, both D_1 and D_2 have directed cycle.*

Proof. Let $D_1 \times D_2$ has a directed cycle. Let $u_1v_1, u_2v_2, \ldots, u_lv_l, u_1v_1$ be a directed cycle. Let us arbitrarily choose any arc from $D_1 \times D_2$. Suppose u_iv_j $\to u_kv_l$, then $u_i \to u_k$ in D_1 and $v_j \to v_l$ in D_2. This is true for all arcs in the