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Edge-Graceful Labelings of Connected Graphs

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Abstract

Let G be a connected edge-graceful (p, q)-graph with $q = kp + r$, where k is an integer and $0 \leq r < p$. In this paper, we prove that every edge-graceful labeling f of G induces $[(k+1)!]^r[k!]^{p-r}$ number of edge-graceful labelings of G.

Keywords: edge-graceful labeling, edge label, vertex label

1 Introduction

Rosa[2] introduced the concept of graceful labeling as a means of attacking the problem of cyclically decomposing the complete graph into other graphs.

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In 1985, Sheng-Ping Lo[3] introduced a similar concept called edge-graceful labeling.

A (p, q)-graph G is said to be edge-graceful if there exists a bijection $f : E \to \{1, 2, ..., q\}$ such that the induced mapping $f' : V \to \{0, 1, 2, ..., p-1\}$ defined by $f'(v) = \sum_{e=(v,u)\in E} f(e) \ (mod\ p)$ is also a bijection.

A necessary condition for edge-gracefulness of a (p, q)-graph G is $q(q+1) \equiv p(p-1)/2 \ \ (mod\ p)$[3].

A detailed list of edge-graceful graphs is given in Gallian Survey of Graph Labelings[1].

In this paper, we derive new edge-graceful labelings from a known edge-graceful labeling. We first start with edge-graceful trees and unicyclic graphs and then extend for all edge-graceful connected graphs. We also enumerate the induced edge-graceful labelings of connected graphs.

For notational convenience, we denote the set of all edges incident with v as $N'(v)$, for every $v \in V$.

2 Edge-Graceful Labelings of Trees and Unicyclic Graphs

Theorem 2.1 *Let f be an edge-graceful labeling of a tree G with p vertices. Then the labeling F defined by $F(e) = p - f(e)$ is also an edge-graceful labeling.*

Proof. For the induced map F', $F'(v) \equiv \sum_{e\in N'(v)} F(e) \equiv \sum_{e\in N'(v)} (p-f(e)) \ (mod\, p)$ $= p - f'(v)$. As f' is a bijection map with the range $\{0, 1, 2, ..., p-1\}$, so also F'. Hence F is also an edge-graceful labeling of G. □

Theorem 2.2 *Let f be an edge-graceful labeling of an unicyclic graph with p vertices. Then the labeling F defined by* $F(e) = \begin{cases} p - f(e) & \text{if } f(e) < p \\ f(e) & \text{if } f(e) = p \end{cases}$ *is also an edge-graceful labeling.*

Proof. Suppose that the edge (w_1, w_2) has the label p, where $w_1, w_2 \in V$.
Case (i) $v \notin \{w_1, w_2\}$.

For the induced map F', $F'(v) \equiv \sum_{e\in N'(v)} F(e) \equiv \sum_{e\in N'(v)} (p - f(e)) \ \ (mod\ p)$ $= p - f'(v)$.
Case (ii) $v \in \{w_1, w_2\}$.