Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 53 (2016) 319–329 www.elsevier.com/locate/endm

A class of *gcd*-graphs having Perfect State Transfer

Hiranmoy Pal [1]

Department of Mathematics
IIT Guwahati
Guwahati, India

Bikash Bhattacharjya [2]

Department of Mathematics
IIT Guwahati
Guwahati, India

Abstract

Let G be a graph with adjacency matrix A. The transition matrix corresponding to G is defined by $H(t) := \exp(itA)$, $t \in \mathbb{R}$. The graph G is said to have perfect state transfer (PST) from a vertex u to another vertex v, if there exist $\tau \in \mathbb{R}$ such that the uv-th entry of $H(\tau)$ has unit modulus. The graph G is said to be periodic at $\tau \in \mathbb{R}$ if there exist $\gamma \in \mathbb{C}$ with $|\gamma| = 1$ such that $H(\tau) = \gamma I$, where I is the identity matrix. A *gcd*-graph is a Cayley graph over a finite abelian group defined by greatest common divisors. In this paper, we construct classes of *gcd*-graphs having periodicity and perfect state transfer.

Keywords: Perfect state transfer, Cayley Graph, Graph products.

[1] Email: `hiranmoy@iitg.ernet.in`

[2] Email: `b.bikash@iitg.ernet.in`

http://dx.doi.org/10.1016/j.endm.2016.05.027

1 Introduction

Perfect state transfer in quantum communication networks was initially studied by S. Bose [4]. Cayley graphs appear frequently in communication networks. Cayley graphs are very well known class of vertex transitive graphs. A circulant graph is a cayley graph over a cyclic group. In [9], it was shown that a circulant graph is periodic if and only if the graph is integral. Also, in [6], Theorem 6.1 implies that a vertex transitive graph admits PST if the graph is periodic. So it is more likely that a Cayley graph would exhibit PST only when the graph has integral spectrum. We therefore investigate PST on *gcd*-graphs as these Cayley graphs are known to have integral spectrum. Some work has already been done in this direction on integral circulant graphs and cubelike graphs. An integral circulant graph is a *gcd*-graph over a cyclic group. Characterization of circulant graphs having PST is given in [2]. A cubelike graph is a *gcd*-graph over the group $\mathbb{Z}_2 \times \ldots \times \mathbb{Z}_2$. Perfect state transfer on cubelike graphs has been discussed in [3,5]. We, however, find PST in more general class of *gcd*-graphs. We now define *gcd*-graph and restate some relevant results.

Let $(\Gamma, +)$ be a finite abelian group and consider $S \subseteq \Gamma$ with the property that $-S = \{-s : s \in S\} = S$, *i.e,* the set S is symmetric. The Cayley graph over Γ with the connection set S is denoted by $Cay\,(\Gamma, S)$. The graph $Cay(\Gamma, S)$ has the vertex set Γ where two vertices $a, b \in \Gamma$ are adjacent if and only if $a - b \in S$. If the additive identity $0 \in S$ then $Cay\,(\Gamma, S)$ has a loop at each of its vertices. We use the convention that each loop **contributes one** to the corresponding diagonal entry of the adjacency matrix. The following lemma implies that the adjacency matrices of two Cayley graphs, defined over a fixed abelian group (finite), commute.

Lemma 1.1 [1] *Let S and T be symmetric subsets of a group Γ. If $gT = Tg$ for every $g \in \Gamma$, then the adjacency matrices of the Cayley graphs $Cay(\Gamma, S)$ and $Cay(\Gamma, T)$ commute.*

Thus one can simply observe: if Γ is an abelian group then adjacency matrices of any two cayley graphs over Γ commute.

The greatest common divisor of two non-negative integers m, n is denoted by $gcd(m, n)$. We use the convention that $gcd(0, n) = gcd(n, 0) = n$ for every non-negative integer n. Consider two r-tuples of non-negative integers $\mathbf{m} = (m_1, \ldots, m_r)$ and $\mathbf{n} = (n_1, \ldots, n_r)$. For $i = 1, \ldots, r$ suppose $d_i = gcd(m_i, n_i)$ and set $\mathbf{d} = (d_1, \ldots, d_r)$. We define $gcd(\mathbf{m}, \mathbf{n}) = \mathbf{d}$.

The additive group of integers modulo n is denoted by $\mathbb{Z}_n$. Let $(\Gamma, +)$ be