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Game Chromatic Number of Direct Product of Some Families of Graphs

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Abstract

In this paper, we determine the exact values of the game chromatic number of direct product graphs $K_{1,n} \times K_{1,m}$, $K_{m,n} \times K_{a,b}$, $P_n \times K_{1,m}$, $P_2 \times W_n$ and $P_2 \times C_n$.

Keywords: Game chromatic number, Direct product

1 Introduction

Let $G = (V, E)$ be a finite graph and X be a set of colors. The vertex coloring game on G is defined through a two person game. Two players, say Alice and

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Bob, with Alice starting first, alternately color a vertex of G with a color from the color set X so that no two adjacent vertices receive the same color. Alice wins the game if all the vertices of G are colored. Bob wins the game if at any stage of the game, there is an uncolored vertex which is adjacent to vertices of all colors from X. The *game chromatic number*, $\chi_g(G)$, of G is the least number of colors in the color set X for which Alice has a winning strategy in the coloring game on G. This parameter is well defined since Alice always wins if $|X| = |V|$. It is obvious that $\chi(G) \leq \chi_g(G) \leq \triangle(G) + 1$, where $\chi(G)$ is the usual chromatic number of G and $\triangle(G)$ is the maximum degree of G.

The game coloring number was first introduced by Zhu [4] as a tool in the study of the game chromatic number. It is also defined through a two person game, say Alice and Bob, with Alice starting first. The players fix a positive integer k and instead of coloring vertices, they mark an unmarked vertex each turn. Bob wins if at some time some unmarked vertex has k marked neighbours, while Alice wins if this never occurs. The game coloring number of G, denoted by $col_g(G)$, is defined as the least number k for which Alice has a winning strategy in the marking game on the graph G. Clearly, if Alice can win the marking game for some integer k, then she can also win the coloring game with k colors, thus $\chi_g(G) \leq col_g(G) \leq \triangle(G)+1$. Note that for any regular graph G, $col_g(G) = \triangle + 1$.

In 2007, T.Bartnicki, B.Bresar, J.Grytczuk, M.Kovsse, Z.Miechowicz and I.Peterin [1] showed that the game chromatic number of Cartesian product of any two simple graphs G and H is not bounded from above by a function of $\chi_g(G)$ and $\chi_g(H)$ and they also determined the exact values of $\chi_g(P_2 \square P_n)$, $\chi_g(P_2 \square C_n)$ and $\chi_g(P_2 \square K_n)$. In 2008, Zhu [5] found a bound for the game chromatic number of Cartesian product graph of any two graphs G and H in terms of their game coloring number and their acyclic chromatic number. In 2009, Charmaine Sia [3] determined the exact value of $\chi_g(S_m \square P_n)$, $\chi_g(S_m \square C_n)$, $\chi_g(P_2 \square W_n)$ and $\chi_g(P_2 \square K_{m,n})$. In 2012, Charles Dunn [2] showed that the game chromatic number of complete r- partite graph with each partite set of size n, K_{r*n}, is always bounded above by a function of r and they also showed that this upper bound is tight.

2 Direct product of graphs

Definition 2.1 The direct product of two simple graphs G and H, denoted by $G \times H$, has vertex set $V(G) \times V(H)$ and edge set $\{(a, x)(b, y) : ab \in E(G) \text{ and } xy \in E(H)\}$.