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Wirelength of Enhanced Hypercubes into r-Rooted Complete Binary Trees

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Abstract

One of the central issues in designing and evaluating an interconnection network is to find out its ability to execute parallel algorithms developed for one network into another with minimum time delay. Solving the wirelength problem helps in minimizing the time delay in execution. The binary hypercube is one of the most popular interconnection networks since it has a simple structure and is easy to implement. The enhanced hypercube is an important variant of hypercube with a smaller diameter, improvised mean node distance and cost efficiency when compared to a binary hypercube. In this paper we consider the problem of embedding enhanced hypercubes into r-rooted complete binary trees for minimizing the wirelength.

Keywords: embedding, enhanced hypercube, complete binary tree, wirelength.

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1 Introduction

The problem of efficiently implementing parallel algorithms developed for one network into another is being studied as a graph embedding problem. Let $G(V, E)$ and $H(V, E)$ be finite graphs on n nodes known as a guest graph and host graph respectively. Let $G[V']$ be the induced subgraph of G whose vertex set is $V' \subseteq V(G)$ and edge set is the set of those edges of G that have both ends in V'. A graph embedding [6,14,17] of G into H is an ordered pair $\prec f, P_f \succ$ defined as follows:

(i) f is a one-to-one map from $V(G)$ to $V(H)$

(ii) P_f is a one-to-one map from $E(G)$ to $\{P_f(u,v) : P_f(u,v)$ is a path in H between $f(u)$ and $f(v)$ for $(u,v) \in E(G)\}$.

An edge congestion of an embedding $\prec f, P_f \succ$ of G into H is the maximum number of edges of the graph G that are embedded on any single edge of H. Let $EC_{\prec f,P_f\succ}(e)$ denote the number of edges (u,v) of G such that e is in the path $P_f(u,v)$ [4,13]. In other words,

$$EC_{\prec f,P_f\succ}(e) = \left|\{(u,v) \in E(G) : e \in P(u,v)\}\right|.$$

The wirelength or layout [9] of an embedding $\prec f, P_f \succ$ of G into H is given by

$$WL_{\prec f,P_f\succ}(G,H) = \sum_{(u,v)\in E(G)} |P_f(u,v)| = \sum_{e\in E(H)} EC_{\prec f,P_f\succ}(e).$$

The wirelength of G into H is defined as

$$WL(G,H) = \min WL_{\prec f,P_f\succ}(G,H)$$

where the minimum is taken over all embeddings f and P_f of G into H. The wirelength problem is to find an embedding that induces $WL(G,H)$. The problem finds application in VLSI designs, networks for parallel computer systems, biological models dealing with cloning and visual stimuli and structural engineering [10,17].

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