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Star chromatic bounds

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Abstract

A *star coloring* of an undirected graph G is a coloring of the vertices of G such that (i) no two adjacent vertices receive the same color, and (ii) no path on four vertices (not necessarily induced) is bi-colored. The *star chromatic number* of G is the minimum number of colors needed to star color G. In this note, we deduce upper bounds for the star chromatic number in terms of the clique number for some special classes of graphs which are defined by forbidden induced subgraphs.

Keywords: Vertex coloring, star coloring, clique number, graph classes.

1 Introduction

All our graphs are simple, finite and undirected, and we follow [4] for standard notations and terminology. As usual, P_n, C_n, and K_n denote the chordless path, chordless cycle and complete graph on n vertices respectively. If $\mathcal{F}$ is a family of graphs, a graph G is said to be $\mathcal{F}$-free if it contains no induced subgraph isomorphic to any graph in $\mathcal{F}$. For any positive integer k, kG denotes the union of k disjoint graphs each isomorphic to G. For a graph G, G^c denotes the complement of G.

A *proper vertex coloring* (or simply *coloring*) of a graph G is an assignment of colors to the vertices of G such that no two adjacent vertices receive the

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same color. The minimum number of colors required to color G is called the *chromatic number* of G, and is denoted by $\chi(G)$. Vertex coloring of graphs has a vast literature, and several variations of vertex coloring have been introduced and studied by many researchers. We refer to a book by Jensen and Toft [15] for an excellent survey of various graph colorings.

In a proper coloring of G, the set of vertices with a same color is called a *color class*. Obviously, the subgraph induced by the union of two color classes is a bipartite graph. In 1973, Grünbaum [13] proposed several variants of vertex coloring with restrictions on the union of two color classes. Among them, star coloring of graphs has received much attention recently (see [1], [10], [16], [17], [18], [21] and [22]). The interest began when it came to be known that these coloring problems have applications in combinatorial scientific computing. In particular, the star coloring problems correspond to direct schemes for recovery of the Hessian matrices. See [12] for a survey of coloring problems as they relate to sparse derivative matrices.

An *acyclic coloring* of a graph G is a coloring of G such that the union of any two color classes induce a forest, that is, a coloring of G such that no cycle in G is bi-colored. A *star coloring* of a graph G is a coloring of G such that the union of any two color classes induce a star forest, that is, a coloring of G such that no path (not necessarily induced) on four vertices is bi-colored. The *star chromatic number* of G is the minimum number of colors required to star color G, and is denoted by $\chi_s(G)$. Note that a star coloring of G is an acyclic coloring of G. Albertson et al. [1] proved that even if the graph G is planar and bipartite, the problem of deciding whether G has a star coloring with 3 colors is NP-complete. So, we are interested in bounds for χ_s when the graph is restricted to certain natural classes of graphs, and bounding χ_s in terms of other parameters of graphs. Albertson et al. [1] showed that (i) if G is embedded on a surface of genus g, then $\chi_s(G) \leq 5g + 20$, and (ii) $\chi_s(G) \leq 20$ if G is a planar graph. Fertin et al. [10] showed that (i) $\chi_s(G) \leq n+1-\alpha(G)$, where $\alpha(G)$ denotes the independence number of G, (ii) $\chi_s(G) \leq \binom{t+2}{2}$, where t denotes the treewidth of G, and (iii) $\chi_s(G) \leq 21\Delta(G)^{3/2}$, where $\Delta(G)$ denotes the maximum degree of G.

In this note, we are interested in deducing bounds for the star chromatic number of a graph in terms of its clique number. A *clique* in a graph G is a set of vertices that are pairwise adjacent in G. The *clique number* of G, denoted by $\omega(G)$, is the size of a maximum clique in G. Obviously, for any graph G, we have $\chi_s(G) \geq \chi(G) \geq \omega(G)$. Lyons [19] showed that if a graph G is (P_4, C_4)-free, then $\chi_s(G) = \chi(G) = \omega(G)$. However, in general, the difference $\chi_s - \omega$ (also $\chi_s - \chi$) can become arbitrarily large. To see this, consider the