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Lower bounds on the sum choice number of a graph

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Abstract

Given a simple graph $G = (V, E)$ and a function f from V to the positive integers, f is called a *choice function* of G if there is a proper vertex coloring ϕ such that $\phi(v) \in L(v)$ for all $v \in V$, where $L(v)$ is any assignment of $f(v)$ colors to v. The *sum choice number* $\chi_{sc}(G)$ of G is defined to be the minimum of $\sum\limits_{v \in V} f(v)$ over all choice functions f of G. In this paper we provide several new lower bounds on $\chi_{sc}(G)$ in terms of subgraphs of G.

Keywords: Graph, coloring, sum choice number

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1 Introduction and Results

Let $G = (V(G), E(G))$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$, and for every vertex $v \in V(G)$ let $L(v)$ be a set (list) of available colors. The graph G is called *L-colorable* if there is a proper coloring ϕ of the vertices with $\phi(v) \in L(v)$ for all $v \in V(G)$. A function f from the vertex set $V(G)$ of G to the positive integers is called a *choice function* of G and G is said to be *f-list colorable* if G is L-colorable for every list assignment L with $|L(v)| = f(v)$ for all $v \in V(G)$. Set $size(f) = \sum_{v \in V(G)} f(v)$ and define the *sum choice number* $\chi_{sc}(G)$ as minimum of $size(f)$ over all choice functions f of G.

Since $\chi_{sc}(G)$ is additive over the set of components of G, we assume in the following that G is connected.

It is easy to see that $\chi_{sc}(G) \leq |V(G)| + |E(G)|$ for every graph G and that there is a greedy coloring of the vertices of G for the corresponding choice function f and every list assignment L with $|L(v)| = f(v)$ for all $v \in V(G)$ (see, e.g., [1]).

Obviously, if $\chi_{sc}(G) \leq k$ and H is a subgraph of G, then $\chi_{sc}(H) \leq k$. Therefore, this property is a hereditary graph property. This implies that a lower bound on the sum choice number of a connected graph G is $2|V(G)| - 1$, since $\chi_{sc}(G) = 2|V(G)| - 1$ if G is a tree [1]. Moreover, $\chi_{sc}(G) = 2|V(G)|$ if G is a cycle [1]. Further results on sum list colorings can be found, e.g., in [2]-[7]. In this paper we present several improvements of $\chi_{sc}(G) \geq 2|V(G)| - 1$ for a connected graph G.

It is natural to pose the question whether it would be possible to obtain information on $\chi_{sc}(G)$ by considering subgraphs of G. A result of this kind is given in Theorem 1.1.

Theorem 1.1 [3] *Let G be a connected graph with blocks $G_1, \cdots, G_k$. Then* $\chi_{sc}(G) = \sum_{j=1}^{k} \chi_{sc}(G_j) - k + 1$.

If G is 2-connected, then G is a block itself and the statement of Theorem 1.1 is trivial in this case. To our knowledge, there is no reasonable result on $\chi_{sc}(G)$ involving arbitrary subgraphs (not necessarily blocks) of G except of trivial observations such as $\chi_{sc}(G) \geq \chi_{sc}(F) + \chi_{sc}(H)$ for any two vertex-disjoint subgraphs F and H of G. In Theorem 1.3, we will show how this obvious inequality can be strengthened.

Our first result is the following Theorem 1.2.