Available online at www.sciencedirect.com
ScienceDirect
Electronic Notes in DISCRETE MATHEMATICS
Electronic Notes in Discrete Mathematics 53 (2016) 465–472
www.elsevier.com/locate/endm

Minimizing the Number of Exceptional Edges in Cellular Manufacturing Problem

S. Arumugam [1] J. Saral [2]

National Centre for Advanced Research in Discrete Mathematics
Kalasalingam University
Anand Nagar, Krishnankoil-626126, India.

A. Somasundaram [3]

Department of Mathematics
BITS, Pilani-Dubai
Dubai International Academic City, UAE

Abstract

The input for a cellular manufacturing problem consists of a set X of m machines, a set Y of p parts and an $m \times p$ matrix $A = (a_{ij})$, where $a_{ij} = 1$ or 0 according as the part p_j is processed on the machine m_i. This data can be represented as a bipartite graph with bipartition X, Y where m_i is joined to p_j if $a_{ij} = 1$. Given a partition π of $V(G)$ into k subsets $V_1, V_2, \ldots, V_k$ such that $|V_i| \geq 2$ and the induced subgraph $\langle V_i \rangle$ is connected, any edge of G with one end in V_i and other end in V_j with $i \neq j$, is called an exceptional edges. The cellular manufacturing problem is to find a partition π with minimum number of exceptional edge. In this paper we determine this number for the subdivision graph of $K_n, K_{m,n}$ and the wheel W_n.

Keywords: Cellular manufacturing problem, part grouping, bipartite graph, subdivision graph, exceptional edge.

http://dx.doi.org/10.1016/j.endm.2016.05.040
1571-0653/

1 Introduction

Cellular manufacturing is an application of the principles of group technology in manufacturing. The input for a cellular manufacturing problem consists of a set X of m machines, a set Y of p parts and an $m \times p$ matrix $A = (a_{ij})$, where $a_{ij} = 1$ or 0 according as the part p_j is processed on the machine m_i. Those parts which require a similar manufacturing process are grouped into a family, called a part family. Given a part family, a group of machines is identified for manufacturing the parts of the family and the part family along with the corresponding group of machines is called a cell. Thus a cell is a small scale, well-defined production unit within a large factory, which has the responsibility for producing a family of parts. Cellular manufacturing problem is to design cells in such a way that some measure of performance is optimized. We confine ourselves to the problem of minimizing the number of part movements from one cell to another cell. Cell formation problem in cellular manufacturing system is an NP-hard problem [4]. Many authors have proposed several approaches for this problem such as mathematical programming [1], neural network [2], graph-theoretic approach [3], genetic algorithm [5], Boolean matrix approach [6], and clustering approach [7].

The machine-part incidence matrix $A = (a_{ij})$ of a cellular manufacturing problem can be represented as a bipartite graph $G = (V, E)$ where $V = X \cup Y$ and a machine $m_i \in X$ is joined to part $p_j \in Y$ if $a_{ij} = 1$. Let $\{G_1, G_2, \ldots, G_k\}$ be a set of connected subgraphs of G such that $\{V(G_1), V(G_2), \ldots, V(G_k)\}$ forms a partition of $V(G)$. Then $\pi = \{V(G_1), V(G_2), \ldots, V(G_k)\}$ is called a k-cell partition of G. Let $\beta(G, \pi)$ denote the number of edges in G with one end in $V(G_i)$ and other end in $V(G_j)$. Thus $\beta(G, \pi)$ denotes the total number of part movements from one cell $V(G_i)$ to another cell $V(G_j)$ for $i \neq j$. An edge of G with one end in $V(G_i)$ and other end in $V(G_j)$ is called an exceptional edge with respect to the partition π. Let $\beta(G, k) = min_\pi \beta(G, \pi)$ where the minimum is taken over all k-cell partitions π of G. Thus $\beta(G, k)$ gives the minimum number of exceptional edges for the given cellular manufacturing problem.

Let $G = (V, E)$ be a graph with $V = \{1, 2, \ldots, n\}$. Let $S(G)$ denote the subdivision graph of G, which is obtained from G by subdividing each edge of G exactly once. We denote by $\{i, j\}$ the vertex subdividing the edge $\{i, j\}$ where $i < j$. Clearly $S(G)$ is a bipartite graph with bipartition V and $\{\{i, j\}$:

[1] Email: s.arumugam.klu@gmail.com
[2] Email: saral2184@gmail.com
[3] Email: asomasundaram@bits-dubai.ac.ae