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Electronic Notes in DISCRETE MATHEMATICS
Electronic Notes in Discrete Mathematics 52 (2016) 5–12
www.elsevier.com/locate/endm

Mathematical Programming with Stochastic Equilibrium Constraints applied to Optimal Last-mile Delivery Services

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Abstract

In e-commerce business, the delivery of products is a crucial part for the success of an e-shop. An efficient delivery system should offer various services and predict customers behaviour. The latter are influenced by the price of a delivery service, but also by its quality (perceived through congestion effect induced by customers' choices). In this study, we introduce a bi-level model to optimize a delivery system. At the upper level, the provider control services' tariffs. At the lower level, users react by choosing their delivery service according to a disutility function which incorporates the provider tariff and the congestion effects. We model the customers' reaction using stochastic user equilibrium (SUE). We also present a sensitivity analysis for the SUE that gives explicit expression of the derivatives of customers distribution with respect to services' tariffs. Based on a local search that exploits the derivatives information, a new heuristic algorithm for a delivery services pricing problem is developed and compared to others existing methods.

Keywords: E-commerce, stochastic user equilibrium, bi-level programming, heuristics.

[1] This work is supported by the French National Research Agency (ANR-Agence Nationale de la Recherche) and is part of project RESPET ANR TTD 2011.

http://dx.doi.org/10.1016/j.endm.2016.03.002

1 Introduction

In many contexts, optimizing a strategic system in which decision makers take decision selfishly on their own, is a challenging problem. This type of problem appears in many contexts like transportation networks, communication networks, economy, etc.. In this paper, we present a mathematical programming approach for solving a bi-level program with stochastic equilibrium constraint for a services' design problem. Our study is motivated by a problem of last-mile delivery services design for on-line retail [9]. After making their purchase, customers choose a delivery service offered by the provider. Each customer decides selfishly the best choice for him. Customers interact all together through congestion effects that are modeled considering queueing metrics. This type of interactive systems with large number of players competing into queueing systems is well overviewed in [8]. We are faced to a stochastic user equilibrium (SUE) problem that defines the lower level of our bi-level problem. At the upper level, the provider has to solve the services' design problem taking into account the equilibrium constraint. This problem appears difficult analytically as the cost functions are not simple and also there is no closed form expression of the dependence between the variables of the provider and the resulting customers' equilibrium. In the following we describe the delivery services and the customer' choice process in section 2. For determining the customer's equilibrium, we consider the method of successive averages (MSA) [14]. In section 3, the services' design problem is addressed and a sensitivity analysis for the SUE is given. We propose an efficient heuristic that is compared to two literature heuristics in section 4, along with suggestions of future works.

2 Delivery services and customers' choice

We assume that each customer chooses one service among a set $\mathcal{J}$ of services. The total number of services proposed is $|\mathcal{J}| = J$. We consider two families or types of services: delivery at home (DH) and delivery at warehouse (WH). The disutility (cost) of service j perceived by a customer depends on the tariff A_j set by the provider, and on the quality of service (evaluated on the congestion effect) induced by other customers' choices. We denote by λ the global customers arrival rate per unit of time. Particularly, we assume that this arrival process follows a Poisson process which means that the expected number of customers, at each time unit, is equal to λ. Each arriving customer makes a decision about his service and we denote by p_j the fraction of customers choosing service $j \in \mathcal{J}$. Note that we have $\sum_{j\in\mathcal{J}} p_j = 1$. The general form of