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A Compact Linearisation of Euclidean Single Allocation Hub Location Problems

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Abstract

Hub location problems are strategic network planning problems. They formalise the challenge of mutually exchanging shipments between a large set of depots. The aim is to choose a set of hubs (out of a given set of possible hubs) and connect every depot to a hub so that the total transport costs for exchanging shipments between the depots are minimised. In classical hub location problems, the unit cost for transport between hubs is proportional to the distance between the hubs. Often these distances are Euclidean distances: Then it is possible to replace the quadratic cost term for hub-hub-transport in the objective function by a linear term and a set of linear inequalities. The resulting model can be solved by a row generation scheme. The strength of the method is shown by solving all AP instances to optimality.

Keywords: Hub Location, Euclidean Distance, Row Generation

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1 Introduction

O'Kelly [5] introduced the Uncapacitated Single Allocation p-Hub Median Problem (USApHMP) in 1987: A set of p hubs is chosen from n possible hub locations and every depot is connected to exactly one hub. The depots mutually exchange shipments: A shipment is sent from the source depot to the assigned hub, then to the assigned hub of the sink depot and then finally to the sink depot. The aim is to choose hubs and assignments with minimal overall transport costs. We state the problem in the original quadratic form:

Consider a complete graph $G = (V, E)$ with $|V| = n$. Each node $i \in V$ of the graph corresponds to origins, destinations and possible hub locations. Let C_{ij} be the transport cost per unit of flow from node i to node j, and W_{ij} be the amount of flow from node i to node j (the *shipment* from i to j). The cost per unit of flow for each path $i \to k \to l \to j$ from an origin node i to a destination node j which passes hubs k and l respectively, is $\beta_1 C_{ik} + \alpha C_{kl} + \beta_2 C_{lj}$, where β_1, α, and β_2 are the collection, transfer and distribution costs respectively.

We define the binary variable x_{ik} to indicate the allocation of node i to the hub located at node k. If node i is assigned to itself, then node i is a hub. To ease the argumentation in the following sections, we define

$$d_{kl} = \alpha C_{kl} \qquad\qquad K_{ik} = \beta_1 C_{ik} \sum_{j \in V} W_{ij} + \beta_2 C_{ki} \sum_{j \in V} W_{ji}.$$

The quadratic $0-1$ formulation of the USApHMP is stated as follows:

$$\min \quad \sum_{i \in V} \sum_{k \in V} K_{ik} x_{ik} + \sum_{i \in V} \sum_{j \in V} \sum_{k \in V} \sum_{l \in V} W_{ij} d_{kl} \cdot x_{ik} x_{jl}$$

$$\text{s.t.} \quad \sum_{k \in V} x_{ik} = 1 \qquad \forall i \in V \tag{1}$$

$$x_{ik} \leq x_{kk} \qquad \forall i, k \in V \tag{2}$$

$$\sum_{k \in V} x_{kk} = p \tag{3}$$

$$x_{ik} \in \{0, 1\} \qquad \forall i, k \in V. \tag{4}$$

Constraints (1) indicate that node i is allocated to precisely one hub node. The inequalities (2) make sure that a node i can only be allocated to a hub node. Constraints (3) force the number of selected hubs to be p.