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# Grid spanners with low forwarding index for energy efficient networks ⋆

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**Abstract**

A routing $R$ of a connected graph $G$ is a collection that contains simple paths connecting every ordered pair of vertices in $G$. The *edge-forwarding index with respect to* $R$ (or simply the forwarding index with respect to $R$) $\pi(G, R)$ of $G$ is the maximum number of paths in $R$ passing through any edge of $G$. The *forwarding index* $\pi(G)$ of $G$ is the minimum $\pi(G, R)$ over all routings $R$'s of $G$. This parameter has been studied for different graph classes [12], [1], [5], [4]. Motivated by energy efficiency, we look, for different numbers of edges, at the best spanning graphs of a square grid, namely those with a low forwarding index.

*Keywords:* spanning subgraphs, forwarding index, energy saving, routing, grid.

## 1 Introduction

A routing $R$ of a given connected graph $G$ of order $N$ is a collection of $N(N-1)$ simple paths connecting every ordered pair of vertices of $G$. The routing $R$ induces on every edge $e$ a *load* that is the number of paths going through $e$. The *edge-forwarding index* (or simply the forwarding index) $\pi(G, R)$ of $G$ with respect to

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$R$ is the maximum number of paths in $R$ passing through any edge of $G$. It corresponds to the maximum load over all edges of the graph when $R$ is used. Therefore, it is important to find routings minimizing this index. The forwarding index $\pi(G)$ of $G$ is the minimum $\pi(G, R)$ over all routings $R$'s of $G$. This parameter has been studied for different graph classes (examples can be found in [1], [5], [4]) and this survey [12] gives a global view on the known results.

We call a connected spanning subgraph of a graph $G$, a *spanner* of $G$. More precisely, it is a connected subgraph that has the same set of vertices as $G$. Our goal is to find, for a given bound on the number of edges, the best spanner of $G$, namely the one with the minimum forwarding index. The problem can also be viewed as: for a given bound $U$ on the forwarding index, find a spanner $F$ of $G$ with minimum number of edges such that $\pi(F) \leq U$.

Knowing how to solve this problem is very interesting in practice for network operators willing to reduce the energy consumed by their networks. In fact, most of the network links consume a constant energy independently of the amount of traffic they are flowing [2], [11]. Therefore, it was proposed to reduce the energy used by the network links by turning some of them off, or more conveniently, putting them into an idle mode. Outside the rush hours, several studies [3], [10], [6], [7], [8], show that a good choice of the links to turn off can lead to significant energy savings, while keeping the same communication quality. In the case where the throughputs from every node to every other node are of the same order, and where the capacities also lie in the same small range, a good choice of those links is reduced to the problem of finding spanners of the network with low forwarding indices.

In this paper, we consider the case in which the initial graph is a square grid. We consider the asymptotic case with $n$ large. We have two main contributions.

On one side, we know that the forwarding index of the $n \times n$ grid $G_n$ is $\frac{n^3}{2}$, see Proposition 1.1 [6]. $G_n$ has $2(n-1)^2 \sim 2n^2$ edges. An important remark is that the load of the edges is lower in the corner than in the middle of the grid. Using that, we show that we can build spanners of $G_n$ with much fewer edges (only $13/18 \approx 72\%$ of the edges) and the same forwarding indices as $G_n$. We show that the proposed spanners are close to optimum in the sense that we prove that it is impossible to build spanners with fewer than $4/3n^2$ edges ($66\%$ of the edges).

On the other side, the smallest possible spanner of the $n \times n$ grid $G_n$ is a spanning tree. The forwarding index of the best spanning tree is asymptotically $\frac{3n^4}{8}$, see Proposition 1.2 [6]. When we add edges and consider spanners with a larger number of edges, the load on the edges decreases, and so does the forwarding index. In this paper, we study how the forwarding index decreases, when we increase the number of edges. The following table summarizes our results: