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Lower Bounding Techniques for DSATUR-based Branch and Bound

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Abstract

Given an undirected graph, the Vertex Coloring Problem (VCP) consists of assigning a color to each vertex of the graph such that two adjacent vertices do not share the same color and the total number of colors is minimized. DSATUR-based Branch-and-Bound is a well-known exact algorithm for the VCP. One of its main drawbacks is that a lower bound (equal to the size of a maximal clique) is computed once at the root of the branching scheme and it is never updated during the execution of the algorithm. In this article, we show how to update the lower bound and we compare the efficiency of several lower bounding techniques.

Keywords: Graph Coloring, DSATUR, Branch and Bound.

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1 Introduction

The *Vertex Coloring Problem* (VCP) is one of the basic problems in graph theory with application in many areas, including: scheduling, timetabling, register allocation, frequency assignment, communication networks and many others. Given an undirected graph $G = (V, E)$ with $|V|$ vertices and $|E|$ edges, a *stable set* of G is a subset of non-connected vertices and a *clique* of G is a subset of fully connected vertices. The *stability number* $\alpha(G)$ is the size of the maximum stable set in G. The *clique number* $\omega(G)$ is the size of the maximum clique in G. A *coloring* C of G is a partition of V into k non empty stable sets: $C = \{V_1, \ldots, V_k\}$. All vertices belonging to V_i are colored with the same color i. The *chromatic number* of G, denoted by $\chi(G)$, is the minimum number of stable sets (or equivalently colors) in a coloring of G. The VCP is the problem of determining the chromatic number of G. A *partial coloring* of G is a partition of $\tilde{V} \subset V$ into $\tilde{k}$ stable sets (i.e., the number of colors): $\tilde{C} = \{\tilde{V}_1, \ldots, \tilde{V}_{\tilde{k}}\}$. The remaining vertices $V \setminus \tilde{V}$ are non-colored. Given a partial coloring $\tilde{C}$ of G, we denote by $\chi_{\tilde{C}}(G)$ the chromatic number of G partially colored by the partial coloring $\tilde{C}$.

The VCP has received a large amount of attention in the last decades. A complete literature review goes beyond the scope of this paper. We refer the reader to Malaguti and Toth [5] for a complete survey on the topic. It is worth mentioning that exact algorithms based on Column Generation and Branch-and-Price (B&P) are particularly effective for "hard" VCP instances, e.g. [3], [4], [6], [7] and [8]. For random graphs instead DSATUR-based Branch and Bound has proven to outperform B&P see [10]. In this paper we study several lower bounding techniques to improve the performances of DSATUR-based Branch and Bound (see [1,10,12]). A second contribution of this paper is to compute and compare the values of a set of lower bounds for the VCP: the clique number, the Lovász Theta number and an upper bound on the stability number of an auxiliary graph (see Section 3). Many theoretical studies have been done on this topic but, to the best of our knowledge, none have experimentally compared the values of those bounds.

2 DSATUR-based Branch and Bound

In this section, we describe the exact implicit enumeration algorithm based on DSATUR, called *DSATUR-based Branch and Bound* (DSATUR-B&B). DSATUR (see Brélaz [1])) is a greedy heuristic algorithm where each vertex $u \in V$ is iteratively colored with a feasible color. Given a (partial) coloring $\tilde{C}$,

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