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A strong symmetric formulation for the Min-degree Constrained Minimum Spanning Tree Problem

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Abstract

Given an integer $K \geq 3$ and an edge weighted undirected graph G, the Min-Degree Constrained Minimum Spanning Tree Problem (MDMST) asks for a minimum cost spanning tree of G, such that every non-leaf vertex has degree at least K. We introduce a new reformulation for MDMST that consists of splitting spanning trees into two parts: a backbone tree spanning only the non-leaf vertices and an assignment of the leaves to non-leaf vertices. The backbone tree is modelled with a lifted version of

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generalized subtour breaking constraints (GSECs). Our computational results show that we can go beyond the strongest MDMST Linear Programming lower bounds in the literature. In addition, a Branch-and-cut algorithm based on our new lower bounds seems to be competitive with the best MDMST exact approach, despite the lack of a primal heuristic. As a by product, new best lower bounds are provided for several unsolved MDMST instances.

Keywords: Min-degree minimum spanning tree, Branch-and-cut, valid inequalities

1 Introduction

Given a connected and undirected graph $G = (V, E)$ and an integer $K \geq 3$, a spanning tree $T = (V, E_T)$ of G is *min-degree constrained* if every non-leaf vertex has degree at least K in the tree. Assuming that weights $\{c_e : e \in E\}$ are assigned to the edges of G, the cost of T is $\sum_{e \in E_T} c_e$. The Min-Degree Constrained Minimum Spanning Tree Problem (MDMST) asks for a min-degree spanning tree of G with minimum cost. MDMST was shown to be NP-hard if $3 \leq K \leq \lfloor \frac{n}{2} \rfloor$ [2,3].

Various solution approaches were proposed since Andrade et al. [3] introduced MDMST. On the metaheuristic side, we can quote the VNS procedure in [9]. Branch-and-bound (BB) and Branch-and-cut (BC) algorithms were also investigated, each of them being based on different formulations for the spanning tree part of feasible solutions [1,3,6,8]. One drawback of the directed cutset formulation in [6,8] is that it provides Linear Programming (LP) bounds that are not symmetrical with respect to the vertex chosen to play the role of root in the spanning arborescences that represent MDMST solutions. Aiming to overcome the lack of symmetry, Martinez and Cunha [8] also proposed the application of the *reformulation by intersection technique* [4] to the directed spanning tree model, obtaining the strongest known formulation at that time. Later, Martinez and Cunha [7] introduced a parallel Lagrangian heuristic, also based on the reformulation by intersection.

In this paper, we introduce a new reformulation for MDMST. It splits min-degree spanning trees into two parts: a backbone tree spanning only the non-leaf vertices and an assignment of the leaves to non-leaf vertices. The

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