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Minimum cost paths over dynamic networks [1]

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Abstract

Flows over time problems relate to finding optimal flows over a capacitated network where transit times on network arcs are explicitly considered. In this paper we study the problem of determining a minimum cost source-destination path where the cost of one arc depends on the time taken to travel from s to that particular arc along the path. We provide a computational complexity characterization for this problem and an exact solution algorithm based on an enumeration scheme on the corresponding time expanded network.

Keywords: Flows Over Time, Dynamic Networks, Computational Complexity.

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1 Introduction

Flows over time (or dynamic flows) model problems where arcs' traversal time is not negligible, such as water distribution systems, street and air traffic problems or material handling in manufacturing environments. In this paper we focus on a special dynamic version of the shortest path problem, a fundamental and widely studied network flow problem which can be solved efficiently with powerful algorithms such as, for instance, A^* (see, e.g., [8]).

Since the seminal paper by Orda and Rom [9], different versions of shortest path problems on dynamic networks have been addressed in the literature (see, for instance, [7]). In this paper we consider an additional issue and take into account a time-dependent cost associated with each arc (u, v) depending on the arrival time at node u: We want to determine a minimum cost (s, r)-path. More precisely, we study the *Minimum Cost Path over Time* (MCPT) problem, which is defined as follows:

- *given* an integer time horizon (or makespan) $T \in \mathbb{Z}_+$, a digraph $G = (N, A)$ with travel time $t_a \in \mathbb{Z}_+$ and cost function $c_a : \{1, \dots, T\} \longrightarrow \mathbb{Z}_+$, for all $a \in A$;
- *find* a simple path P from an origin $s \in N$ to a destination $r \in N$
- *such that* the total travel time fits the time-budget, i.e. $\sum_{a \in P} t_a \leq T$ and, its cost

$$c(P) := \sum_{a \in P} c_a(t_{a,P}) \tag{1}$$

  is minimum, where $t_{a,P}$ is the time needed to reach arc a (i.e., if $a = (u, v)$, the time to travel from s to u on path P).

Hereafter, a path P minimizing the cost function (1) will be also referred to as a *cheapest path*.

Many problems similar to MCPT have been addressed in the literature, see for instance [3,10]. Together with network design problems [1], finding a cheapest path is especially motivated by emergency evacuation planning, where arc costs represent adverse or harmful effects associated with the evacuation process like, for instance, congestion or temporary roads unavailability. Moreover, in [2], it is shown that some special multi-commodity minimum cost

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