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The 1,2-Conjecture for powers of cycles ⋆

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Abstract

A $[k]$-total-weighting ω of a simple graph G is a mapping $\omega\colon V(G) \cup E(G) \to \{1, \ldots, k\}$. A $[k]$-total-weighting ω of G is neighbour-distinguishing if, for each pair of adjacent vertices $u, v \in V(G)$, the value $\omega(u) + \sum_{uw \in E(G)} \omega(uw)$ is distinct from $\omega(v) + \sum_{vw \in E(G)} \omega(vw)$. The 1,2-Conjecture states that every simple graph G has a neighbour-distinguishing [2]-total-weighting. In this work, we prove that the 1,2-Conjecture is valid for all powers of cycles.

Keywords: total-weighting, neighbour-distinguishing, 1,2-Conjecture, powers of cycles

1 Introduction

Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. We denote an edge $e \in E(G)$ by uv when u and v are its endpoints. An *element* of G is a vertex or an edge of G. As usual, we denote by $d(v)$ the degree of a vertex $v \in V(G)$. We say that a graph is k-*regular* if all of its vertices have degree k.

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A *vertex-colouring* of a graph G is an assignment φ of colours to the vertices of G. If $\varphi(u) \neq \varphi(v)$ for each pair of adjacent vertices, φ is said to be a *proper vertex-colouring*. An *edge-weighting* of a graph G is an assignment of integer values (*weights*) to the edges of G. Given an edge-weighting of G, a vertex-colouring of G can be obtained in a natural way by defining the *colour* of each vertex as the sum of the weights of its incident edges.

In 1985, G. Chartrand et al. [1] proposed the problem of determining the least positive integer k needed to construct an edge-weighting of a simple graph G with integers from 1 to k such that, for any two vertices u and v of G, the colour of u is different from the colour of v. This problem was later investigated by other authors and some variants were proposed [3, 4, 6, 7]. In particular, M. Karónski et al. [4] asked what is the least positive integer k needed to construct an edge-weighting $\lambda\colon E(G) \to \{1, \ldots, k\}$ of a simple graph G such that adjacent vertices have distinct colours. In this article, M. Karónski et al. conjectured that, for any simple graph G, there exists such an edge-weighting with integers in the set $\{1, 2, 3\}$. This conjecture is known as the *1,2,3-Conjecture* and remains open for arbitrary graphs. Motivated by these edge-weighting problems, J. Przybyło and M. Woźniak [6] defined a closely related weighting called neighbour-distinguishing total-weighting, formally defined as follows.

A *total-weighting* of a graph G is an assignment of weights to the edges and to the vertices of G. A total-weighting $\omega\colon V(G) \cup E(G) \to \{1, \ldots, k\}$ is also called a $[k]$-*total-weighting* of G. Given a total-weighting ω of G, let $c_\omega(v) := \omega(v) + \sum_{uv \in E(G)} \omega(uv)$ define the *colour* of each vertex $v \in V(G)$. We say that a total-weighting ω is *neighbour-distinguishing* if, for each pair of adjacent vertices $u, v \in V(G)$, $c_\omega(u) \neq c_\omega(v)$. If such a weighting exists, we say that G *admits* a neighbour-distinguishing total-weighting. The smallest k for which G admits a neighbour-distinguishing $[k]$-total-weighting is denoted by $\tau(G)$. In their article, the authors posed the following conjecture:

Conjecture 1 (J. Przybyło and M. Woźniak [6]) *If G is a simple graph, then G admits a neighbour-distinguishing $[2]$-total-weighting.*

Conjecture 1 is known as the *1,2-Conjecture* and has been approached for some classes of graphs [5, 6]. Some upper bounds for $\tau(G)$ have also been determined [2, 5, 6]. In fact, a major breakthrough in the determination of an upper bound to $\tau(G)$ was the result of M. Kalkowski [2], who proved that $\tau(G) \leq 3$ for all graphs. Despite this result, no graph G with $\tau(G) > 2$ is known. In this work, we verify the 1,2-Conjecture for powers of cycles, a suitable class of graphs for dealing with labelling problems.