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Excluding clocks

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Abstract

A clock is a cycle with a vertex that has exactly two neighbors on the cycle. We show that (triangle, cube, clock)-free graphs of girth at least 9 always contain a vertex of degree 2, partially answering to a conjecture of Trotignon. As a second result, we show that the class of clock-free graphs is χ-bounded by $\max(4, \omega(G))$.

1 Introduction

A graph G *contains* a graph F if an induced subgraph of G is isomorphic to F. A graph G is *F-free* if G does not contain F. For a set of graphs $\mathcal{F}$, G is *$\mathcal{F}$-free* if it is F-free for every $F \in \mathcal{F}$.

In this article, we study graphs without a *clock*, a graph made of an induced cycle C and a vertex x not in C with exactly two neighbors in C. We denote such a clock by (x, C) and call C the *rim* and x the *center* of the clock. We are interested in proving the existence of a vertex of degree at most 2 in clock-free graphs as well as bound their chromatic number.

Previous work

Clock-free graphs generalize several studied classes of graphs as we explain now.

A *propeller* is a graph made of an induced cycle C and a vertex u not in C that has at least 2 neighbors in C. Observe that the class of propeller-free

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graphs is a proper subclass of clock-free graphs. In [1], it is proved that every propeller-free graph has a vertex of degree at most 2.

A *theta* is a graph made of three internally vertex-disjoint paths $P_1 = a \dots b$, $P_2 = a \dots b$, $P_3 = a \dots b$ of length at least 2 and such that no edges exist between the paths except the three edges incident to a and the three edges incident to b. Observe that a theta with a path of length exactly 2 is a clock, but a clock does not need to be a theta: if the rim is a triangle, then it is not a theta. But (triangle, theta)-free graphs form a proper subclass of clock-free graphs. In [3], it is proved that, if a (triangle, theta)-free graph contains a cube, then either it is the cube or it has a clique cutset and that every (triangle, cube, theta)-free graph has a vertex of degree at most 2.

Analogously, we have the two following structural results on clock-free graphs:

Lemma 1.1 [4] *If a clock-free graph G contains a triangle, then either G is a clique or G has a clique cutset.*

Lemma 1.2 *If a clock-free graph G contains a cube, then either G is a cube, or G has a clique cutset.*

This leads to the following conjecture proposed by Trotignon that generalizes the two above results.

Conjecture 1.3 *[4] A (triangle, cube, clock)-free graph contains a vertex of degree at most 2.*

Our results

Our main result is the proof of Conjecture 1.3 for graphs of girth at least 9. In order to do so, we prove (see Section 2) a stronger version of it for 2-connected graphs that easily implies the 1-connected case.

Theorem 1.4 *Every 2-connected clock-free graph with girth at least 9 contains at least two vertices of degree 2.*

We observe that Conjecture 1.3 implies that the chromatic number of a (triangle,clock)-free graph is at most 3. So, since a clock-free graph that contains a triangle is either a clique or has a clique cutset, Conjecture 1.3 implies that $\chi(G) \leq \max(3, \omega(G))$ and this would be best possible. In Section 3, we prove that if G is a clock-free graph, then $\chi(G) \leq \max(4, \omega(G))$.

Notations

[4] Due to space restriction, some of the proofs are omitted.

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