Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 50 (2015) 121–126 www.elsevier.com/locate/endm

Inapproximability of the lid-chromatic number

Nicolas Martins [a,2] Rudini Sampaio [a,2]

[a] *Dept. Computação, Universidade Federal do Ceará, Fortaleza, Brazil*

Abstract

A lid-coloring (locally identifying coloring) of a graph is a proper coloring such that, for any edge uv where u and v have distinct closed neighborhoods, the set of colors used on vertices of the closed neighborhoods of u and v are also distinct. In this paper we obtain a relation between lid-coloring and a variation, called strong lid-coloring. With this, we obtain linear time algorithms to calculate the lid-chromatic number for some classes of graphs with few P_4's. We also prove that the lid-chromatic number is $O(n^{1/2-\varepsilon})$-inapproximable in polinomial time for every $\varepsilon > 0$, unless P=NP.

Keywords: Locally identifying coloring, inapproximability, cographs, P_4-sparse graphs.

1 Introduction

Given a vertex v of a graph G, let $N(v)$ be the set of neighbors of v and let $N[v] = N(v) \cup \{v\}$ be the closed neighborhood of v. A vertex is *universal* if $N[v] = V(G)$. Given a proper coloring c of $V(G)$, let $c(S)$ for any $S \subseteq V(G)$ be the set of colors in the vertices of S and let the *code* of a vertex v be $c(N[v])$ (the colors in the closed neighborhood of v).

A *lid-coloring* c of G is a proper coloring of $V(G)$ such that any two adjacent vertices with distinct closed neighborhoods have distinct codes. That is, for every edge uv, if $N[u] \neq N[v]$, then $c(N[u]) \neq c(N[v])$. The *lid-chromatic number* $\chi_{lid}(G)$ is the minimum integer k such that there is a lid-coloring of G using k colors.

Locally identifying colorings were introduced in 2012 by Esperet et al. [7] and are related to distinguishing colorings [1,2,3] and identifying codes [11].

[1] Email: {nicolasam,rudini}@lia.ufc.br

[2] The authors are partially supported by CNPq, CAPES and FUNCAP.

http://dx.doi.org/10.1016/j.endm.2015.07.021

In 2012, Esperet et al. proved that, in a bipartite graph G, $\chi_{lid}(G) \geq 4$ but deciding if $\chi_{lid}(G) \leq 3$ is NP-complete. They also obtain polinomial algorithms for trees and for cographs with bounded clique number [7]. In this paper, we obtain linear time algorithms for general cographs and P_4-sparse graphs.

In 2012, Foucaud et al. proved that $\chi_{lid}(G) \leq 2\Delta(G)^2 - 3\Delta(G) + 3$ [8], answering a question of [7]. In 2013, Gonalves et al. proved that every planar graph has a lid-coloring with 1280 colors [9] answering another question of [7].

Esperet et al. also introduced the concept of strong lid-coloring. A *slid-coloring* c of G is a lid-coloring such that, for any non-universal vertex v, $c(N[v]) \neq c(V(G))$. In other words, if $c(N[v]) = c(V(G))$ (v "sees" all colors), then v is universal. The *slid-chromatic number* $\widetilde{\chi_{lid}}(G)$ is the minimum k such that G has a slid-coloring with k colors.

In Section 2, we prove that $\chi_{lid}(G)$ is $O(n^{1/2-\varepsilon})$-inapproximable in polynomial time for all $\varepsilon > 0$, unless P=NP. To the best of our knowledge, it is the first inapproximability result for the lid-chromatic number. In Section 3, we prove that $\chi_{lid}(G) \leq \widetilde{\chi_{lid}}(G) \leq \chi_{lid}(G)+1$. In Section 4, we obtain linear time algorithms to determine $\chi_{lid}(G)$ for cographs and P_4-sparse graphs. There will be clear the importance of the computation of $\widetilde{\chi_{lid}}(G)$ for the computation of $\chi_{lid}(G)$. This improves the result of Esperet et. al. (see Section 6 of [7]) for cographs with bounded clique number.

2 Inapproximability of χ_{lid} and $\widetilde{\chi_{lid}}$

Theorem 2.1 *The lid-chromatic and the slid-chromatic numbers are $O(n^{1/2-\varepsilon})$-inapproximable in polynomial time for every $\varepsilon > 0$, unless P=NP.*

Proof. [Sketch] We obtain an approximation preserving reduction (see the appendix for the details) from the Minimum Proper Coloring Problem. The chromatic number $\chi(G)$ is the minimum k such that G has a proper coloring with k colors. We will consider instances G such that $\chi(G) \geq 3$.

Given a graph G, we construct a graph G' using the following procedure. We begin the construction of G' with the same vertex set of G and with an empty edge set. For each vertex $v \in V(G)$, we add a vertex v' and make v' adjacent to v in G'. For each edge $e = uv \in E(G)$, we add two vertices u_e and v_e and three edges $u_e u$, $u_e v_e$ and $v_e v$ in G'.

It is not difficult to see that any lid-coloring c' of G' induces a proper coloring c of G, since, if $c'(u) = c'(v)$ for an edge $e = uv$ of G, then the codes of u_e and v_e are the same, equal to $c'(N[u_e]) = c'(N[v_e]) = \{c'(v), c'(u_e), c'(v_e)\}$, a contradiction since they are adjacent. Then $\chi(G) \leq \chi_{lid}(G') \leq \widetilde{\chi_{lid}}(G')$.