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Structural characterization and decomposition for cographs-$(2, 1)$ and $(1, 2)$: a natural generalization of threshold graphs [1]

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Abstract

A cograph is a graph without induced paths of length 4. A graph G is $(2, 1)$ if its vertex set can be partitioned into at most 2 independent sets and 1 clique. Cographs-(k, ℓ) have already a characterization by forbidden subgraphs, but no structural characterization is known, except for cographs-$(1, 1)$, i.e threshold graphs. In this paper, we present a structural characterization and a decomposition theorem for cographs-$(2, 1)$ and, consequently, for cographs-$(1, 2)$, leading to linear time recognition algorithms for both classes.

Keywords: Cographs-$(2, 1)$,cographs-$(1, 2)$, structural characterization, decomposition, threshold.

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1 Introduction

Perfect graphs atract a lot of attention in graph theory as well as partition problems. In [1,2,3], Brandstädt *et al.* defined a special class named (k, ℓ)-*graphs*, i.e, graphs whose vertex set can be partitioned into at most k independent sets and ℓ cliques: a generalization of split graphs, which can de described as $(1, 1)$-graphs. Moreover, they proved that the recognition problem for this class of graphs is NP-complete for k or ℓ at least 3 and polynomial, otherwise. In this work we restrict this recognition problem to a subclass of perfect graphs: cographs.

Definition 1.1 *[4] A cograph can be defined recursively as follows:*

(i) *The trivial graph K_1 is a cograph;*

(ii) *If $G_1, G_2, \ldots, G_p$ are cographs, then $G_1 \cup G_2 \cup \ldots \cup G_p$ is a cograph,*

(iii) *If G is a cograph, then $\bar{G}$ is a cograph.*

There are some equivalent forms of characterizing a cograph [4], but one of the well known is the characterization by forbidden subgraphs.

Theorem 1.2 *[4]*
A cograph is a graph without induced P_4, i.e. induced paths of length 4.

Corneil in 1985 [5], presented the first, but not the only one, linear time algorithm to recognize cographs [6,7].

Threshold graphs are a special case of cographs and split graphs. More formally, a graph is a threshold graph if and only if it is both a cograph and a split graph. Introduced by Chvátal and Hammer in 1977 [8], Theorem 1.3 characterizes them.

Theorem 1.3 *[8] For every graph G, the following three conditions are equivalent:*

(i) *G is threshold;*

(ii) *G has no induced subgraph isomorphic to $2K_2, P_4$ or C_4;*

(iii) *There is an ordering $v_1, v_2, \ldots, v_n$ of vertices of G and a partition of $\{v_1, v_2, \ldots v_n\}$ into disjoint subsets P and Q such that:*
- *Every $v_j \in P$ is adjacent to all vertices v_i with $i < j$,*
- *Every $v_j \in Q$ is adjacent to none of the vertices v_i with $i < j$.*

Thus, threshold graphs can be constructed from a trivial graph K_1 by repeated applications of the following two operations:

(i) Addition of a single isolated vertex to the graph.