

The Online Prize-Collecting Facility Location Problem

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Abstract

In this paper we propose the Online Prize-Collecting Facility Location problem (OPCFL), which is an online version of the Prize-Collecting Facility Location problem (PCFL). The PCFL is a generalization of the Uncapacitated Facility Location problem (FL) in which some clients may be left unconnected by paying a penalty. Another way to think about it is that every client has a prize that can only be collected if it is connected. We give a primal-dual $O(\log n)$ -competitive algorithm for the OPCFL based on previous algorithms for Online Facility Location (OFL) due to Fotakis [3] and Nagarajan and Williamson [9].

Keywords: Online Algorithms, Competitive Analysis, Prize-Collecting Facility Location Problem, Primal-Dual Method.

1 Introduction

In the Facility Location problem (FL), we have a set of clients and a set of facilities in a metric space. Each facility has an associated opening cost. The

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cost of assigning a client to a facility is the distance between them. The goal of the problem is to select a set of facilities to open and to assign clients to open facilities so that the total cost of opening the facilities plus the cost of connecting clients to their assigned facilities is minimized. FL is an NP-complete problem that has been well-studied; several constant ratio approximation algorithms are known for it [10,7,2,6]. It is particularly interesting that several different design techniques, such as LP rounding, primal-dual and local search, are successful at achieving good approximation ratios for this problem.

The online version of FL is the Online Facility Location problem (OFL), in which the clients are revealed one at a time and each one needs to be connected to an open facility before the next one arrives. Throughout this process, no connection can be changed or opened facility can be closed. Algorithms for online problems are analyzed via competitive analysis [1]. An α -competitive algorithm returns a solution whose cost is within a factor of α of the cost of an optimal solution to the corresponding offline problem; α is called the competitive ratio of the algorithm. There are randomized and deterministic $O(\log n)$ -competitive algorithms known for the OFL [8,4,3,9,5], where n is the number of clients. Moreover, the best lower bound for the competitive ratio of an algorithm for OFL is $\Omega\left(\frac{\log n}{\log \log n}\right)$ [4].

The Prize-Collecting Facility Location problem (PCFL) is a generalization of the FL in which some clients may be left unconnected by paying a penalty. Another way to think about it is that every client has a prize that can only be collected if it is connected. There is a constant ratio approximation algorithm known for PCFL, due to Xu and Xu [11], that combines the primal-dual method with local search techniques.

Our Contributions. We propose the Online Prize-Collecting Facility Location problem (OPCFL), which is an online version of the PCFL. We give a primal-dual $O(\log n)$ -competitive algorithm for the OPCFL based on algorithms for OFL due to Fotakis [3] and Nagarajan and Williamson [9]. Since the OPCFL generalizes the OFL, the lower bound of $\Omega\left(\frac{\log n}{\log \log n}\right)$ applies to it.

2 Preliminaries

In this section we define the Online Prize-Collecting Facility Location problem (OPCFL) and present some notation and definitions that are useful later. Also, we show a linear programming relaxation for the OPCFL.

The OPCFL is a generalization of the OFL in which some clients may be left unconnected by paying a penalty. In general we use i to denote a facility

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