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Electronic Notes in DISCRETE MATHEMATICS
Electronic Notes in Discrete Mathematics 50 (2015) 157–162
www.elsevier.com/locate/endm

Chronological Rectangle Digraphs

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Abstract

Interval graphs admit elegant ordering and structural characterizations. A natural digraph analogue of interval graphs, called chronological interval digraphs, has recently been identified and studied. In this paper, we introduce the class of chronological rectangle digraphs, a generalization of chronological interval digraphs. We show that several properties of chronological interval digraphs extend to this larger class of digraphs. We also show that chronological rectangle digraphs are all weakly clustered. Finally we give an ordering characterization of chronological rectangle digraphs, akin to that of chronological interval digraphs, which generalizes the usual ordering characterization of interval graphs.

Keywords: Interval graph, boxicity, Ferrers dimension, chronological interval digraph, chronological rectangle digraph, structure, characterization.

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http://dx.doi.org/10.1016/j.endm.2015.07.027
1571-0653/

1 Introduction

Interval graphs are perhaps the most studied class of graphs in structural graph theory. Numerous characterizations along with linear time recognition algorithms for interval graphs have been obtained, cf. e.g. [1,3].

Interval graphs have been generalized in different ways. In [7], Roberts introduced the concept of boxicity. A graph G is of *boxicity at most* k if it is an intersection graph of axis-parallel boxes in k-dimensional space. Interval graphs are just the graphs of boxicity one. Graphs of boxicity at most two (i.e., intersection graphs of axis-parallel rectangles in the plane) have attracted a great deal of attention, cf. e.g. [5,6]. Despite a characterization given in [6], recognizing graphs of boxicity at most two is an NP-complete problem, cf. [5].

A digraph analogue of interval graphs (under the name of interval digraphs) was pioneered in [8]; these are the digraphs where each vertex v is associated with an ordered pair (I_v, J_v) of intervals such that uv is an arc if and only if $I_u \cap J_v \neq \emptyset$. Interval digraphs can be recognized in polynomial time but their structure is not well-understood. Another natural alternative digraph analogue of interval graphs was recently proposed in [2]. A digraph G is a *chronological interval digraph* if there exists a family of closed intervals $I_v, v \in V(G)$ such that uv is an arc of G if and only if I_u contains the left endpoint of I_v. (Equivalently, uv is an arc of G if and only if $I_u \cap I_v \neq \emptyset$ and the left endpoint of I_u is not greater than the left endpoint of I_v.) Comparing to interval digraphs, chronological interval digraphs are closer to interval graphs; they admit ordering and structural characterizations similar to those of interval graphs along with a linear time recognition algorithm [2]. These most attractive aspects of interval graphs are unfortunately missing for interval digraphs.

The purpose of this paper is to introduce a new class of digraphs which we call chronological rectangle digraphs. A digraph G is a *chronological rectangle digraph* if there exists a family of axis-parallel closed rectangles $R_v, v \in V(G)$ such that uv is an arc of G if and only if R_u contains the lower left corner of R_v. The family of rectangles is called a *rectangle model* for the graph G. Since each rectangle contains its own lower left corner, every chronological rectangle digraph is reflexive. For the same reason, interval graphs and chronological interval digraphs are all reflexive. As intervals can be viewed as degenerated rectangles, every chronological interval digraph is a chronological rectangle digraph. Projecting each rectangle in the rectangle model for G to the x- and y-axes respectively we obtain interval models for two chronological interval digraphs G_1 and G_2 on the same vertex set $V(G)$. It is easy see that uv is an