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Optimal Edge Fault-Tolerant Bijective Embedding of a Complete Graph over a Cycle.

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Abstract

An embedding of a guest graph G over a host graph H is an injective map Φ from the vertices of G to the vertices of H and a *routing map* ρ, which associates every edge $e = xy$ in G to a $\Phi(x)$-$\Phi(y)$ path $\rho(e)$ in H. Given an edge f in H the number of edges e in G such that f belongs to $\rho(e)$ is the (edge) congestion $cong(f)$ of f. The length of $\rho(e)$ is called the dilatation $dil(e)$ of e. The sum of all the dilatations is the cost of the embedding. The removal of an edge f of H gives rise to a *surviving graph* G_f, consisting of the guest graph without those edges that cross f, i.e., $G_f = G - \{e : f \in \rho(e)\}$. Given n and b, we are facing the problem of finding a minimum cost embedding Φ of a graph G with n vertices over the cycle C_n, such that for any surviving graph G_f, there is an embedding of the complete graph K_n over G_f whose congestions are not greater than b. This work presents a lower bound for the optimal cost of such problem and a family of embeddings that match this bound over a broad range of combinations of n and b.

Keywords: Graph Theory, Multilayer Networks, Routing.

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1 Introduction

The concept of 2-edge-connectivity is widely required in network design problems. It guarantees the existence of a path between any two nodes when any edge faults, which was sufficient for many historical applications and still is for some of them. As a common characteristic, these problems do not integrate demands and capacities; they are only concerned with connectivity, disregarding which or how many paths use a common link. Nowadays the stress caused by the growth of demands compromises these premisses. The multicommodity flow problem is a well-suited approach for some of these cases, but it doesn't fit well for others, particularly for designing telecommunications backbones.

Real-world telecommunications networks are structured into layers where: subterranean/submarine conduits are used to channelize optical fibers, over which optical-transport-networks are deployed in turn (see [5,3]). Internet traffic flows over IP/MPLS networks, whose nodes are linked by high-speed optical connections (see [4,7,2]), and so on. Besides, there is another key aspect to consider, the topology of each layer is usually richer than that of the underneath one. Therefore, a fault on a single physical link (a conduit) may cause a huge number of faults upwards, which could collapse the services supported by the upper layers. The concept of embedding (see [8]) gives us an appropriate and unified framework to express such a structure of nested levels, combining demands and capacities with connectivity constraints.

The problem this article is inspired by is called *Free Routing Protection Multi-Overlay Resilient Network Design Problem*, or FRP-MORNDP for short. The FRP-MORNDP is quite general and involves real-world networks. For having a historical and technical perspective of FRP-MORNDP, we invite the reader to overview [6], where the author proves its NP-Hardness and develops heuristic approaches to find solutions for real-world instances. The present work tackles down the problem for the family of instances where physical topologies are cycles with n vertices and the remaining parameters (distances, demands) equal to 1 except for b, the unique logical links' capacity (see Section 2 for a precise definition). We call this subfamily of problems CYCLE-FRP-MORNDP.

Although simpler, CYCLE-FRP-MORNDP is far from being easy. Besides it has many practical applications because physical networks are structured as compositions of cycles. It is easy to see that there is no solution for $b = 1$ and that when $b \geq \lceil n/2 \rceil \lfloor n/2 \rfloor$, the optimal solution is C_n itself, with the shortest-path routing. In [1], optimal solutions for odd n and $b = 2$ were found. Besides, in that work, it is proved that if n is even, the optimal solutions exist only for $b \geq 3$. The graphs in Fig. 1 are the computer-aided constructions of CYCLE-