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# Efficient and Perfect domination on circular-arc graphs

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**Abstract**

Given a graph $G = (V, E)$, a *perfect dominating set* is a subset of vertices $V' \subseteq V(G)$ such that each vertex $v \in V(G) \setminus V'$ is dominated by exactly one vertex $v' \in V'$. An *efficient dominating set* is a perfect dominating set $V'$ where $V'$ is also an independent set. These problems are usually posed in terms of edges instead of vertices. Both problems, either for the vertex or edge variant, remains NP-Hard, even when restricted to certain graphs families. We study both variants of the problems for the circular-arc graphs, and show efficient algorithms for all of them.

*Keywords:* Efficient Domination, Perfect Domination, Circular-Arc graphs

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# 1 Introduction

Given a graph $G = (V, E)$, a **perfect dominating set** is a subset of vertices $V' \subseteq V(G)$ such that each vertex $v \in V(G) \setminus V'$ is dominated by exactly one vertex $v' \in V'$. An **efficient dominating set** is a perfect vertex dominating set $V'$ where $V'$ is also an independent set. Every graph $G$ contains a perfect dominating set, for instance, take $V(G)$. But not every graph contains an efficient vertex dominating set. These problems consists in searching the sets with minimum number of vertices. All of them are NP-hard, even when restricted to certain graph families. The weighted version of these problems, where each vertex $v$ has a weight assigned $\omega(v)$, consists on finding a perfect vertex dominating set where the sum of the weights is minimum. We denote these problems as Minimum Weighted Perfect Vertex Domination (MWPVD), Minimum Weighted Efficient Vertex Domination (MWEVD). We denote the edge-versions of these problems as Minimum Weight Perfect Edge Domination (MWPED) and Minimum Weight Efficient Edge Domination (MWEED). Efficient edge dominating sets are also known as dominating induced matchings, and denoted as DIM's. Note that for these edge-versions the dominating set consists of edges instead of vertices, hence the weights are on the edges, and the adjacency of two edges is defined as two edges that shares a vertex. We say a *pendant* vertex (also known as *leaf*) is one whose degree is exactly one. In this paper we show results for the weighted perfect domination problem, and for the efficient domination problem, restricted to circular-arc graphs. The proofs and details of this paper can be found in [6].

# 2 Circular-Arc graphs

The following definitions and results come from [4]

Given a circular-arc model $\mathcal{M} = (C, \mathcal{A})$ where $\mathcal{A} = \{A_1 = (s_1, t_1), \ldots, A_n = (s_n, t_n)\}$, two points $p, p' \in C$ are equivalent if $\mathcal{A}(p) = \mathcal{A}(p')$. The $2n$ extreme points from the $n$ arcs of $\mathcal{A}$ divide the circle $C$ in $2n$ segments of the following types: (i) $(s_i, t_j)$ (ii) $[t_i, t_j)$ (iii) $(s_i, s_j]$ (iv) $[t_i, s_j]$. We say the segments of type (i) are *intersection segments*. It is easy to see that all points inside one of the $2n$ segments are equivalent.

**Corollary 2.1** *[4]There are at most $2n$ distinct $\mathcal{A}(p)$.*

**Lemma 2.2** *[4] Given a CA model $\mathcal{M} = (C, \mathcal{A})$, if there are no two or three arcs of $\mathcal{A}$ that covers the entire circle $C$ then $\mathcal{M}$ is an HCA model.*

We consider the four variants of the mentioned problems for circular-arc