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On second iterated clique graphs that are also third iterated clique graphs

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Abstract

Iterated clique graphs arise when the clique operator is applied to a graph more than once. Determining whether a graph is a clique graph or an iterated clique graph is usually a difficult task. The fact that being a clique graph and being an iterated clique graph are not equivalent things has been proved recently. However, it is still unknown whether the classes of second iterated clique graphs and third iterated clique graphs are the same. In this work we find classes of graphs, defined by means of conditions on the clique size and the structure of the clique intersections, whose second iterated clique graphs are also third iterated clique graphs.

Keywords: Clique graph, clique operator, iterated clique graph, line graph

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1 Introduction

Let G be a simple graph. A *clique* of G is a maximal set of pairwise adjacent vertices of G. The set of cliques of G is denoted by $\mathcal{C}(G)$.

The *clique graph* of G, or $K(G)$, is the graph whose vertices are the cliques of G, in a way that two different cliques C and C' of G are adjacent in $K(G)$ if and only if $C \cap C' \neq \emptyset$. Denote the class of all graphs by $\mathcal{G}$. The function $K : \mathcal{G} \to \mathcal{G}$ that assigns to each graph its clique graph is called the *clique operator*. G is said to be a clique graph if $G = K(H)$, for some graph H. Thus, $K(\mathcal{G})$ is the class of clique graphs.

Given $n > 1$, the function K^n is the composition of the clique operator with itself n times. The *n-th iterated clique graph* of G is the graph $K^n(G)$. G is an *n-th iterated clique graph* if $G = K^n(H)$, for some graph H. Thus, $K^n(\mathcal{G})$ is the class of n-th iterated clique graphs.

Determining whether a given graph is a clique graph is usually a complicated task. Actually, the clique graph recognition problem is NP-complete [1]. The complexity of recognizing iterated clique graphs has not been established yet. Furthermore, very little is known about the relationship between the classes $K(\mathcal{G})$, $K^2(\mathcal{G})$, $K^3(\mathcal{G})$ ... and their differences.

It is now known that $K(\mathcal{G})$ and $K^2(\mathcal{G})$ are different because the clique graph of the octahedron is not a second iterated clique graph [2]. However, no other example of a considerably different graph in $K(\mathcal{G}) \setminus K^2(\mathcal{G})$ is known and it is unknown whether $K^n(\mathcal{G}) = K^{n+1}(\mathcal{G})$ for $n > 2$.

This work is focused on comparing $K^2(\mathcal{G})$ and $K^3(\mathcal{G})$. We prove that, for every graph G with cliques of size at most 3, $K^2(G)$ is a graph in $K^3(\mathcal{G})$. The proof of this result involves line graphs. Afterwards, we identify some patterns in the proof to attain a more general result that allows cliques of size larger than 3 in the graphs that we consider. As a result of this work, we get a wide variety of graphs in $K^2(\mathcal{G}) \cap K^3(\mathcal{G})$, in a good starting point for proving that $K^2(\mathcal{G}) = K^3(\mathcal{G})$, in case that this is really true. If false, it could help in the refinement of the search for a graph in $K^2(\mathcal{G}) \setminus K^3(\mathcal{G})$, since many possible counterexamples are discarded.

2 Results

As it was said in the Introduction, it is known that $K(\mathcal{G}) \neq K^2(\mathcal{G})$ because the clique graph of the octahedron is a graph in $K(\mathcal{G}) \setminus K^2(\mathcal{G})$ [2]. The octahedron and its clique graph are members of a special class of graphs, called *octahedral graphs*.