Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 50 (2015) 373–378 www.elsevier.com/locate/endm

On Rotational Symmetries of Drawings of Coherent Periodic Graphs

Tetsuya Araki [1]

National Institute of Informatics
Principles of Informatics Research Division
Tokyo, Japan

Norie Fu [1]

JST, ERATO, Kawarabayashi Large Graph Project
National Institute of Informatics, Tokyo, Japan

Abstract

Periodic graphs are used to model crystals. The symmetry of crystal structure is one of the important subjects. A symmetric drawing of the periodic graphs reveals the symmetry of a crystal structure. In this paper, we present a necessary condition so that a drawing, where the edges can be drawn by general curves, of a coherent periodic graph can admit the translational symmetry and the rotational symmetry.

Keywords: Graph Drawing, Symmetry, Periodic Graph

1 Introduction

For the qualitative analysis and the description of crystal structures, graphs obtained from crystals are often used. From this perspective, one of the important problems on these graphs to determine the combinatorial symmetry group, or the maximal symmetry group that an embedding of such a graph into a space [9]. Since the symmetries in an embedding of a graph clarify the

[1] Email: {araki,funori}@nii.ac.jp. We are grateful to Dr. Nakagawa for his suggestion of the problem

http://dx.doi.org/10.1016/j.endm.2015.07.062
1571-0653/

structures of the graph visually, there are many previous works on algorithms for symmetric embeddings. On the other hand, not every graph has a symmetric embedding. Lubiw [7] and Manning [8] showed that it is NP-complete to decide whether a given finite graph has a symmetric embedding.

To show the impossibility of a symmetric embedding, necessary conditions for the existence of symmetric embeddings are useful. In this paper, we investigate such necessary conditions about *periodic graphs* and *drawings*. Periodic graphs are a special kind of infinite graphs, that is defined from a given finite directed graph, called a *static graph*. A d-dimensional periodic graph is constructed from a static graph as the follows: copy the vertices of the static graph to each cell of the $\mathbb{Z}^d$-lattice, and connect them periodically subject to the rule indicated by the edges of it. Periodic graphs are often used as a model of crystals [1]. A drawing is a general kind of graph embedding where edges can be general curves and can intersect. Deza and Shtogrin [3] suggested a necessary condition for the existence of a drawing that admits both the translational symmetry and the rotational symmetry of a given ℓ_1-rigid periodic graph. For 2-dimensional periodic graphs, this condition be verified by using an algorithm for computing ℓ_1-embedding of a 2-dim periodic graphs [5].

In this paper, we deal with a special class of 2-dimensional periodic graph, called *coherent* periodic graphs. It is shown that all ℓ_1-rigid periodic graphs are also coherent [4]. We first show that the target graph class in this paper properly contains the class of ℓ_1-rigid graphs. A drawing of a periodic graph generated by a static graph $\mathcal{G}$ is *consistent with* $\mathcal{G}$, if it is obtained by putting a drawing of $\mathcal{G}$ periodically on the plane. The main result is a necessary condition for the existence of a consistent drawing with a given static graph that admits both the translational symmetry and the rotational symmetry. An algorithm for verifying this condition is also given.

2 Preliminaries

Definition 2.1 For $n \in \mathbb{N}$, let $[n] := \{1, \ldots, n\}$. A *static graph* is a pair of a vertex set $\mathcal{V} = [n]$, and a set of directed edges endowed with vectors $\mathcal{E} = \{e_1, \ldots, e_m\} \subseteq (\mathcal{V} \times \mathcal{V}) \times \mathbb{Z}^d$.

Definition 2.2 For a static graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, the *periodic graph* $G = (V, E)$ *generated by* $\mathcal{G}$ is an infinite graph such that $V = \mathcal{V} \times \mathbb{Z}^d$, and $E = \{((i, \mathbf{h}), (j, \mathbf{h}+\mathbf{g})) : \mathbf{h} \in \mathbb{Z}^d, ((i, j), \mathbf{g}) \in \mathcal{E}\} \subset V \times V$.

By definition, a periodic graph is a directed graph. In this paper, we ignore the directions of the edges, and assume that d is always 2. Note that different two static graphs can generate the same periodic graph.