

A hierarchical network formation model

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Abstract

We present a network formation model based on a particularly interesting class of networks in social settings, where individuals' positions are determined according to a topic-based or hierarchical taxonomy. In this game-theoretic model, players are located in the leaves of a complete b -ary tree as the seed network with the objective of minimizing their collective distances to others in the network. In the grid-based model of Even-Dar and Kearns [3], they demonstrate the existence of small diameter networks with the threshold of $\alpha = 2$ where the cost of a new link depends on the distance between the two endpoints to the power of α . We show the appearance of small diameter equilibrium networks with the threshold of $\alpha = 1/4$ in the hierarchical or tree-based networks. Moreover, the general set of equilibrium networks in our model are guaranteed to exist and they are pairwise Nash stable with transfers [2].

Keywords: Network formation, Hierarchical networks, Linking game with transfers.

1 Introduction

The role of network structures in determining the outcome of many social and economic settings has captured increasing attention in recent years. In particular, different researches have demonstrated this significant role in the wide range of problems [4]. As a result, it is crucial to study the formation process of social and economic networks and to know their characteristics, as

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there is a frequent appearance of certain properties such as small diameter and large clustering coefficient in naturally occurring networks [7]. Stochastic models are the first class of efforts that have been proposed to study network formation. On the other hand, a growing body of works in strategic or game-theoretic network formation³ considers the underlying network structure as a set of self-interested individuals and their connections. In a previous effort [3], Even-Dar and Kearns proposed a model that starts from a $\sqrt{n} \times \sqrt{n}$ grid network. Players' objective is to minimize their collective distances to all other players. Forming a link is costly at the fixed price of the initial grid distance between the endpoint players of that link to the power of α . Also, equilibrium analysis is relaxed from Nash equilibrium to link stability. Link stability implies that the network is stable under single link unilateral deviations.⁴ They showed the appearance of small diameter link stable networks within the threshold of $\alpha = 2$. Further, Atabati and Farzad [1] expand this result by showing the existence of a small diameter in the general set of equilibrium networks under the new assumption of *dynamic link-pricing* that better represents the dynamic nature of network formation.⁵

Hierarchical networks are known as credible social structures based on observations in social sciences [8]. In particular, previous studies [5,8] have analyzed the hierarchical networks by applying stochastic models. However, less is known from the strategic perspective. We aim to fill this gap by presenting our game-theoretic hierarchical network formation model. The rest of this paper is organized as follows. In Section 2, we explain our model. We then provide our analysis on the diameter of equilibrium networks in Section 3 and the conclusion in Section 4.

2 The model

Let $N = \{1, \dots, n\}$ be the set of n players being located at the leaves or the lowest level of a complete b -ary tree, denoted by G_0 as the starting or seed network. Typically in these networks, the distance of each two players is defined to be the height of their lowest common ancestor in the tree.⁶ Note that in the beginning, there is no additional link between the players in the

³ See [4] for a comprehensive survey in this topic.

⁴ This is as opposed to Nash, which allows for arbitrary link unilateral deviations.

⁵ Dynamic link-pricing updates the applied distances of each pair of players in the related link-prices from the current network rather than sticking with the initial grid distances.

⁶ Considering a total of n players in the network, the upper bound for the distance between two players is $\log_b n$.

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