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Upper Bound for Failure Risk in Networks

Piotr Chołda[1,2] Krzysztof Rusek[3] Piotr Guzik[4]

AGH University of Science and Technology, Krakow, Poland

Abstract

A method for calculating the upper bound of total penalty that may be paid by an operator if the services provided are interrupted due to network failures is presented. The level of penalty is expressed as a commonly accepted business risk measure, and correlations between failures influencing various services are taken into account.

Keywords: Compensation policy, continuity, network design, risk quantification

1 Introduction: Business-related Risk Quantification

Service Level Agreements (SLAs) define the desired values of parameters related to the services provided, including reliability in presence of network failures. Penalties for not meeting these requirements may also be agreed and form the basis for calculating monetary impact to quantify business/financial risk. A probabilistic *risk measure* ρ is used to delimit the *level* and *variability* of the penalties. Here, we assess the value of a risk measure for the total penalties paid by the network operator during a given time interval.

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[2] Email: piotr.cholda@agh.edu.pl

[3] Email: krusek@agh.edu.pl

[4] Email: guzik@kt.agh.edu.pl

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The way a penalty is defined as a function of the technical reliability parameter is known as the *compensation policy* [1]. The typical `Avail` policy is to define the *availability*, i.e. the fraction of time when the service is operating, over a time interval, meaning that cumulative downtime per interval is the basis of the penalty. It has been shown that this approach is not relevant to some highly demanding network services (e.g. real-time control traffic) [2]. In such cases, the `Cont` policy based on *continuity* rather than availability is relevant, and the number of failures should be the basis for compensation.

Quantile risk measures known as Value-at-Risk (VaR) or its variations are most commonly applied. These are popular in the investment management or financial sectors and are also proposed to be used in networking [3]. Let X be the level of penalties to be paid in an interval. X pertains to a single service or a whole network. If $P_X(x) = \Pr\{X \leq x\}$ is the cumulative distribution function of X, VaR is defined as the maximum penalty with a given confidence level η: $VaR_\eta = \sup\{x : \Pr\{X \leq x\} \leq \eta\} = P_X^{-1}(\eta)$. P_X distributions can be of various types; nevertheless it is common practice in the investment sector to base the VaR-related calculations on normal distributions [4]. Although VaR is most commonly used, it lacks the property of subadditivity [5]. Presence of subadditivity justifies pressure on portfolio diversification, which decreases the risk, and which is a phenomenon observed in the markets. Subadditivity is possessed by the Conditional Value-at-Risk ($CVaR_\eta$): the mean value of penalties, if they exceed VaR_η. It is known that $CVaR$ provides also more reliable data than VaR, especially if distribution of X has a heavy tail [5].

2 Upper Bound for Risk

To be able to provide the general results, our analysis follows assumptions taken in the seminal work [6]. The calculation algorithm is as follows: (I) Determine the failure/repair data for all the unreliable network components i: λ_i, μ_i. Define compensation policies for penalties p for each service s, find routing for all the services and the resulting macrocomponents. (II) Construct the Markov chains for all the involved macrocomponents using Eq. (3); calculate means and variances of up- and downtimes for all the macrocomponents. (III) Calculate the policy-related values for all the components on the basis of the properties given in Eq. 4. (IV) Find the covariance matrix of $\mathbf{X}$ using Eq. (2) and the mean value of $\mathbf{X}$ with Eq. (1). (V) Calculate ρ on the basis of the whole normal distribution of p parametrized by $\mathrm{E}[\mathbf{X}]$ and $\mathrm{Cov}[\mathbf{X}]$.

The following mathematical framework enables us to express the compensation policy if it is consistent among all the services. A network is represented