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Towards Optimal Embedding of an Arbitrary Tree in a Graceful Tree

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Abstract

A function f is called a graceful labeling of a graph G with m edges, if f is an injective function from $V(G)$ to $\{0, 1, 2, \cdots, m\}$ such that when every edge uv is assigned the edge label $|f(u) - f(v)|$, then the resulting edge labels are distinct. A graph which admits a graceful labeling is called a graceful graph. The popular and notorious Graceful Tree Conjecture: "All trees are graceful", remains unsettled over four decades inspite of many rigorous attempts and investigations. Inspired by the interesting result of Acharya et al [1], in this paper we show that every tree can be embedded in a graceful tree. Practically, we observe that any tree with m edges can be embedded in a graceful tree with less than $2m$ edges and we discuss a related open problem towards settling the Graceful Tree Conjecture through the embedding approach.

Keywords: Graceful Tree,Graceful Tree Conjecture,Graceful Tree Embedding,Graceful Labeling,Graph Labeling.

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1 Introduction

All the graphs considered in this paper are finite and simple graphs. The terms which are not defined here can be refered from [7]. In 1963, Ringel posed his celebrated conjecture, popularly called Ringel Conjecture [6], which states that, K_{2n+1}, the complete graph on $2n+1$ vertices can be decomposed into $2n+1$ isomorphic copies of a given tree with n edges. For an exhaustive survey on graceful trees refer the dynamic survey by Gallian [2]. Inspired by the interesting result of Acharya et al [1], in this paper we show that every tree can be embedded in a graceful tree. Practically, we observe that any tree with m edges can be embedded in a graceful tree with less than $2m$ edges and we discuss a related open problem towards settling the Graceful Tree Conjecture through the embedding approach.

2 Main Result

In this section, we show that every tree can be embedded in a graceful tree through Graceful Tree Embedding Algorithm. Graceful Tree Embedding Algorithm takes arbitrary tree T as its input and it runs Labeling Algorithm on the tree T, obtains its output T' and then it extends T' to a gracefl tree T^*. First we present Bipartition Algorithm that defines a bipartition of vertex set of given arbitrary input tree T, then this bipartition of vertices of T will be used in Labeling Algorithm.

Bipartition Algorithm

Input: Any arbitrary tree T with m edges and diameter d

Step 1:

Identify a longest path P of length d of T.

Step 1.1:

Arrange the vertices of the longest path P of T from left to right and label them $u_0, u_1, \cdots, u_d$.

Step 2:

For each branch that comes out from the vertex u_i of the longest path P, for i, $0 \leq i \leq d$, visit the vertices of the branch at the vertex u_i in pre-order traversal by considering the branch at the vertex u_i as a rooted tree with u_i as its root. Denote the vertices of the branch at u_i that are visited in the pre-order traversal by $u_{i,1}, u_{i,2}, \cdots, u_{i,t_i}$, where t_i is the number of vertices in the branch at the vertex u_i, for $0 \leq i \leq d$.