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Gracefully Cultivating Trees on a Cycle

Ramon M. Figueroa-Centeno [1]

Mathematics Department,
University of Hawaii at Hilo, 200 W. Kawili St. Hilo, HI 96720, USA.

Rikio Ichishima [2]

College of Humanities and Sciences,
Nihon University, 3-25-40 Sakurajosui Setagaya-ku, Tokyo 156-8550, Japan.

Francesc A. Muntaner-Batle [3]

Graph Theory and Applications Research Group, School of Electrical Engineering and Computer Science, Faculty of Engineering and Built Environment,
The University of Newcastle, NSW 2308, Australia.

Akito Oshima [4]

Graph Theory and Applications Research Group, School of Electrical Engineering and Computer Science, Faculty of Engineering and Built Environment,
The University of Newcastle, NSW 2308, Australia.

Abstract

A graph G of size q is graceful if there exists an injective function $f : V(G) \to \{0, 1, \ldots, q\}$ such that each $uv \in E(G)$ is labeled $|f(u) - f(v)|$ and the resulting edge labels are distinct. Truszczyński conjectured that all unicyclic graphs except the cycle C_n, where $n \equiv 1$ or $2 \pmod 4$, are graceful. In this paper, we present two

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methods to construct certain graceful unicyclic graphs when the length of cycles are congruent to 0 or 3 (mod 4).

Keywords: graceful labellings, unicyclic graphs, Truszczyński's conjecture

1 Introduction

We start by referring the reader to Chartrand and Lesniak [3] for all the definitions and notations not provided in this paper. The graphs we consider are finite and simple, that is, there are no loops and multiple edges.

A graph G of size q is called *graceful* if there exists an injective function $f : V(G) \to \{0, 1, \ldots, q\}$ such that each $uv \in E(G)$ is labeled $|f(u) - f(v)|$ and the resulting edge labels are distinct. Such a function is called a *graceful labeling*. A graceful labeling f is called an *α-valuation* if there exists an integer λ (called the *critical value*) so that $\min\{f(u), f(v)\} \leq \lambda < \max\{f(u), f(v)\}$ for each $uv \in E(G)$.

The investigation of graceful labelings of cycle-related graphs have been a major focus of attention for many years. Rosa first showed in [9] that the cycle C_n is graceful if and only if $n \equiv 0$ or $3 \pmod 4$. Afterwards, Truszczyński [11] investigated the graceful properties of unicyclic graphs and proved that several classes of such graphs possess graceful labelings. In his work, Truszczyński conjectured that all unicyclic graphs except C_n, where $n \equiv 1$ or $2 \pmod 4$, are graceful.

Several researchers have considered the gracefulness of a particular class of unicyclic graphs. The interested reader is now directed to the surveys of graph labelings by Gallian [6] for other results, which partially resolve the aforementioned conjecture of Truszczyński.

For any integer $n \geq 3$, given a cycle C_n and a family of trees $\mathfrak{T} = \{T_1, T_2, \ldots, T_n\}$, let u_i and v_i, $1 \leq i \leq n$, be the fixed vertices of C_n and T_i, respectively. Then the unicyclic graph, $\mathrm{Amal}(C_n, \mathfrak{T})$, can be obtained from C_n and $\mathfrak{T}$ by amalgamating C_n and T_i at the vertices u_i and v_i, $1 \leq i \leq n$. In this paper, we present two construction methods that generate a graceful labeling of the unicyclic graph $\mathrm{Amal}(C_n, \mathfrak{T})$ from a cycle C_n and a family of trees $\mathfrak{T}$ with α-valuations when $n \equiv 0$ or $3 \pmod 4$.

[1] Email: ramonf@hawaii.edu
[2] Email: ichishima@chs.nihon-u.ac.jp
[3] Email: famb1es@yahoo.es
[4] Email: akitoism@yahoo.co.jp

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