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Graceful digraphs and complete mappings

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Abstract

Bloom and Hsu while extending the graceful labelings of graphs to digraphs, specified the relation between graceful unicycles and complete mappings by establishing the relation of each to a particular class of permutations. We denote $\vec{C_m}(r;m)$ as a digraph with two directed cycles, one with vertices $v_1, v_2, ..., v_{r-1}, v_r, v_{r+1}, ..., v_m$ and another directed cycle with vertices $v_1, v_2^1, ..., v_{r-1}^1, v_r, v_{r+1}^1, ..., v_m^1$ of same length, such that both the directed cycles have v_1 and v_r as the two common vertices (where $m \geq 4$, $3 \leq r \leq m-1$). In this paper we use complete mappings to deduce a partition of Z_n, where $n = 2m+1$ odd and show that the digraph $\vec{C_m}(r;m)$ is graceful.

Keywords: Graceful digraphs, complete mappings, partitions of Z_n.

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1 Introduction

For standard notations and terminologies in graph theory we follow Chartrand and Lesniak [2].

The concept of graceful labeling was introduced by Rosa [9] in the year 1967.

Definition 1.1 An undirected graph with e edges is gracefully labeled if each vertex v is assigned a distinct value $f(v)$ from $\{0, 1, ..., e\}$ in such a way that the set of edge labels equals $\{1, 2, ..., e\}$ when edge uv is labeled by $f(u, v) = |f(u) - f(v)|$. A graph is said to be graceful (undirected) graph if it can be gracefully labeled.

This concept was extended to digraphs by Bloom and Hsu in [1].

Definition 1.2 A digraph D with p vertices and q arcs is labeled by assigning a distinct integer value $g(v)$ from $\{0, 1, 2, ..., q\}$ to each vertex v. The vertex values, in turn, induce a value $g(u, v)$ on each arc (u, v) where $g(u, v) = (g(v) - g(u))\ (mod\ q + 1)$. If the arc values are all distinct, then the labeling is called a graceful labeling of the digraph D.

The following are from [6].

Theorem 1.3 *Let D be a graceful digraph with p vertices and q arcs. Suppose the directed cycle $\vec{C}_m$ is contained in the digraph D. Then the sum of the labels on the arcs of $\vec{C}_m$ is congruent to zero$(mod\ q + 1)$.*

Theorem 1.4 *The directed cycle $\vec{C}_m$ is graceful iff the sum of the elements $1, 2, ..., m$ is congruent to zero$(mod\ m + 1)$ and there exists an arrangement of these elements in a circular way, with the sum of $n(n < m)$ consecutive elements not congruent to zero$(mod\ m + 1)$.*

One can see a detailed study of graph labeling problems given by Gallian in his survey paper [4].

We denote a digraph as $\vec{C}_m(r; m)$, if it consists two directed cycles each of length m, one with vertices $v_1, v_2, ..., v_{r-1}, v_r, v_{r+1}, ..., v_m$ and the other directed cycle with vertices $v_1, v_2^1, ..., v_{r-1}^1, v_r, v_{r+1}^1, ..., v_m^1$ such that both the directed cycles have v_1 and v_r as the two common vertices, where $m \geq 4$, $3 \leq r \leq m-1$.

Here $\vec{C}_m(r; m)$ is viewed as a digraph having two directed cycles each of length m and with two common vertices.

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