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Complete Multipartite Graphs and their Null Set

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Abstract

For every natural number h, a graph G is said to be h-magic if there exists a labelling $l : E(G) \longrightarrow Z_h \setminus \{0\}$ such that the induced vertex set labelling $l^+ : V(G) \longrightarrow Z_h$ defined by

$$l^+(v) = \sum_{uv \in E(G)} l(uv),$$

is a constant map. When this constant is zero, it is said that G admits a zero-sum *h-magic* labelling. The null set of a graph G, denoted by $N(G)$, is the set of all natural numbers $h \in N$ such that G admits an h-zero-sum magic labelling. In 2007,

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E. Salehi determined the null set of complete bipartite graphs. In this paper we generalize this result by obtaining the null set of complete multipartite graphs.

Keywords: magic, zero-sum, null set.

1 Introduction

For an abelian group A, written additively, any mapping $l : E(G) \longrightarrow A \setminus \{0\}$ is called a *labelling*. Given a labelling on the edge set of G, one can introduce a vertex labelling $l^+ : V(G) \longrightarrow A$ by

$$l^+(v) = \sum_{uv \in E(G)} l(uv).$$

A graph G is said to be A-*magic* if there is a labelling $l : E(G) \longrightarrow A \setminus \{0\}$ such that for each vertex v, the sum of labels of all edges incident with v, is a constant; that is, $l^+(v) = c$, for some $c \in A$. In general, a graph G may admit more than one labelling to become A-magic. For example, if $|A| > 2$ and $l : E(G) \longrightarrow A \setminus \{0\}$ is a magic labelling of G with sum c, then $\lambda : E(G) \longrightarrow A - \{0\}$, the inverse labelling of l, define by $\lambda(uv) = -l(uv)$ will provide another magic labelling of G with sum $-c$. We use $K_{n_1,\ldots,n_k}$ to denote the complete multipartite graph with part sizes $n_1, \ldots, n_k$. We denote the complete graph of order n by K_n. Also, if we decompose $E(G)$ into $E(H_1), \ldots, E(H_k)$, then we write $G = H_1 \oplus \cdots \oplus H_k$. Within the mathematical literature, various definition of magic graphs have been introduced. The original concept of an A magic graph is due to Sedlacek [3,4], who defined it to be graph with real-valued edge labelling such that distinct edges have distinct non-negative labels, and the sum of the labels of edges incident with a particular vertex is the same for all vertices. Over the years, there has been a great research interest in graph labelling problems. In fact, many different graph labellings have been introduced in the literature. The interested readers is referred to Wallis' [5] recent monograph on magic graphs. For convenience, a $\mathbb{Z}_h$-magic graph will be referred to as an h-*magic graph*. An h-magic graph G is said to have h-*zero-sum magic labelling* if there is a magic labelling of G in $\mathbb{Z}_h$ that induces a vertex labelling with sum 0. The *null set* of a graph G, denoted by $N(G)$, is the set of all natural numbers $h \in N$ such G admits

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