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The Second Minimum of the Irregularity of Graphs

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Abstract

For a graph G, Albertson [1] has defined the irregularity of G as

$$irr(G) = \sum_{xy \in E(G)} |d_G(x) - d_G(y)|$$

where $d_G(u)$ is the degree of vertex u. Recently, this graph invariant gained interest in chemical graph theory. In this work, we present some new results on the second minimum of the irregularity of graphs.

Keywords: Irregularity, graph.

1 Introduction

The word graph refers to a finite, undirected graph without loops and multiple edges. The vertex and edge sets of G are $V(G)$ and $E(G)$. The irregularity of

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a graph [1] G is defined as:

$$irr(G) = \sum_{xy \in E(G)} |d_G(x) - d_G(y)| \tag{1}$$

where $d_G(x)$ is the degree of vertex x. The irregularity of graph is a graph invariant that some of graph invariants reported in [2-5]. Let $N(x)$ is the set of vertices of G that are adjacent to x. For $x, y, u \in V(G)$, define the sets $N_x, N_x^1, N_x^2, N_{xy}^1$ and N_{xy}^2 of vertices of G:
$N_x = \{u \in V(G) \mid ux \in E(G), d_G(x) > d_G(u)\}$, $n_x = |N_x|$,
$N_x^1 = \{u \in V(G)|ux \in E(G), d_G(x) \geq d_G(u)\}$, $n_x^1 = |N_x^1|$,
$N_x^2 = \{u \in V(G)|ux \in E(G), d_G(x) < d_G(u)\}$, $n_x^2 = |N_x^2|$,
$N_{xy}^1 = \{u \in V(G)|ux \in E(G), d_G(x) > d_G(u), u \neq y\}$, $n_{xy}^1 = |N_{xy}^1|$,
$N_{xy}^2 = \{u \in V(G)|ux \in E(G), d_G(x) \leq d_G(u), u \neq y\}$, $n_{xy}^2 = |N_{xy}^2|$.

2 Main Results

In this section, we express some old results on irregularity of $G + xy$, $G - xy$ and characterize all graphs with irregularity 2 [1].

Lemma 2.1 *If $xy \notin E(G)$ then, $irr(G+xy) = irr(G) + 2\Big(max[d_G(x), d_G(y)] - n_x^2 - n_y^2\Big) = irr(G) + 2\Big(n_x^1 + n_y^1 - min[d_G(x), d_G(y)]\Big)$.*

Lemma 2.2 *If $xy \in E(G)$ then, $irr(G-xy) = irr(G) + 2\Big(min[d_G(x), d_G(y)] - n_{xy}^1 - n_{yx}^1 - 1\Big) = irr(G) + 2\Big(n_{xy}^2 + n_{yx}^2 + 1 - max[d_G(x), d_G(y)]\Big)$.*

Lemma 2.3 *Let G be a graph, then $irr(G) = \sum_{x \in V(G)} (n_x - n_x^2) d_G(x)$.*

Theorem 2.4 *Let G be a graph then $irr(G)$ is only even number.*

Corollary 2.5 *The second minimum of the irregularity of graphs is 2.*

Theorem 2.6 *There are 25 types of connected graphs with irregularity 2.*

Proof. Our main proof will consider three separate cases as follows:
Case 1. Let x and y be distinct nodes of G such that xy an edge of G and $|d_G(x) - d_G(y)| = 2$.
Suppose $xu_1u_2 \cdots u_s y$ is a path in G joining x and y, since $irr(G) = 2$ and $|d_G(x) - d_G(y)| = 2$, then $d_G(x) = d_G(u_1) = d_G(u_2) = \cdots = d_G(u_s) = d_G(y)$. Therefore, G is composed of two separate components G_1 and G_2 that are connected by xy edge, which $x \in V(G_1), y \in V(G_2), G_1 \cup G_2 = G$ and