Physical Communication 3 (2010) 1–18

Contents lists available at ScienceDirect

Physical Communication

journal homepage: www.elsevier.com/locate/phycom

Full length article

Emerging applications of wavelets: A review

Ali N. Akansu [a,*], Wouter A. Serdijn [c], Ivan W. Selesnick [b]

[a] New Jersey Institute of Technology, Department of Electrical & Computer Engineering, University Heights, Newark, NJ 07102, United States
[b] Polytechnic Institute of New York University, Department of Electrical & Computer Engineering, Six MetroTech Center, Brooklyn, NY 11201, United States
[c] Delft University of Technology, Faculty of Electrical Engineering, M & CS, Electronics Research Laboratory, Mekelweg 4, 2628 CD Delft, The Netherlands

ARTICLE INFO

Keywords:
Analog signal processing
Wavelet
Discrete wavelet
Discrete wavelet approximation
Analog wavelet filters
Subband transform
Filter bank
Iterative-thresholding

ABSTRACT

Although most of its popular applications have been in discrete-time signal processing for over two decades, wavelet transform theory offers a methodology to generate continuous-time compact support orthogonal filter banks through the design of discrete-time finite length filter banks with multiple time and frequency resolutions. In this paper, we first highlight inherently built-in approximation errors of discrete-time signal processing techniques employing wavelet transform framework. Then, we present an overview of emerging analog signal processing applications of wavelet transform along with its still active research topics in more matured discrete-time processing applications. It is shown that analog wavelet transform is successfully implemented in biomedical signal processing for design of low-power pacemakers and also in ultra-wideband (UWB) wireless communications. The engineering details of analog circuit implementation for these continuous-time wavelet transform applications are provided for further studies. We expect a flurry of new research and technology development activities in the coming years utilizing still promising and almost untapped analog wavelet transform and multiresolution signal representation techniques.

1. Historical perspective and current status

Multiresolution representation of image and video has always generated an interest in vision research due to the spectral properties and models of the human visual system (HVS). Burt and Adelson proposed their pyramid decomposition algorithm for multiresolution coding of image signals [1]. Their pioneering work opened the gate for a new image coding technique where better frequency localized subband transforms found their use as an alternative to the widely used discrete cosine transform (DCT) coding [2–4]. On the same track, Mallat looked into the continuous-time discrete wavelet transform as a multiresolution signal decomposition technique and its interconnections to discrete-time filter banks in his doctoral dissertation [5].

Independently, Daubechies elegantly formalized the theoretical linkage, first brought up by Mallat, between finite support orthonormal wavelet transform basis and two-band perfect reconstruction quadrature mirror filter (PR-QMF) bank in her seminal paper published in 1988 [6]. While Goupillaud, Grossmann, Morlet and other researchers in Europe made their early contributions, Daubechies' celebrated paper was the most significant starter for wavelet related research activities in the United States [7–9]. The New Jersey Institute of Technology Wavelets Symposium and the NSF CBMS Conference on Wavelets in 1990 [10,11] were the first two technical conferences in the Unites States where Signal Processing, Mathematics and other research communities shared the same venues for interdisciplinary exchanges and cross-fertilization have taken place.

Most of the relevant early work in signal processing field was to show and emphasize the theoretical interconnections and linkages between wavelet and subband transforms proven by Daubechies [6,12]. There were other

* Corresponding author.
E-mail addresses: akansu@njit.edu (A.N. Akansu), w.a.serdijn@tudelft.nl (W.A. Serdijn), selesi@poly.edu (I.W. Selesnick).

1874-4907/$ – see front matter
doi:10.1016/j.phycom.2009.07.001

research contributions that offered filter bank solutions that lead to the design of compactly supported orthonormal wavelet bases. On the other hand, the theory of wavelet transforms were better understood by signal processing engineers and several tutorial papers on the subject published in the literature. A number of well written early research monographs, lecture notes and edited books on wavelet transforms and multiresolution signal processing and filter banks were published [13–15]. Then, there were many good quality papers and books on wavelet and subband transforms, and their applications published in the signal processing literature.

In addition to highlighting some novel implementation and applications of analog wavelet transforms, this review paper will describe wavelet transform approximation errors inherent in discrete-time signal processing and also some of the principle developments in wavelet theory for analog signal processing.

One of the important developments in the construction, design, and implementation of wavelet (multiscale) transforms is the design of geometrically-oriented two-dimensional (and higher) transforms. For example, starting with the steerable pyramid and continuing with the curvelet, contourlet, and shearlet, dual-tree complex wavelet transforms and wave atoms. These are especially important because some of the popular applications of multiscale transform are in image processing, for which these transforms make a substantial difference. In addition, these years have seen many advances in digital filter bank theory, for example in parameterizations, directional filter banks, and others. Additionally, the frequency domain design and FFT-based implementation of wavelet transforms has received new attention in the recent literature, for both one-dimensional and multidimensional non-separable geometrically-oriented transforms, because this approach overcomes limitations of FIR-filter-based designs. For example, using an FFT-implementation, orthonormal dyadic discrete wavelet transforms with symmetric filters and symmetric boundary extensions can be implemented for any signal length, including odd-lengths. In contrast, the conventional orthonormal FIR-convolution implementation cannot be simultaneously orthonormal and symmetric (excepting the Haar transform) and is furthermore usually implemented for signals whose lengths are powers of two.

One of the many active application areas of wavelet transform has been of denoising. The state-of the-art has progressed significantly over the last 20 years. For applications of denoising, the noise is rarely entirely Gaussian nor signal independent. Therefore, signal processing methods that can be applied to realistic scenarios are of continuing interest. Wavelet-based algorithms have also been developed for the problem of deconvolution. Some deconvolution and denoising algorithms can be unified into the framework of *iterative thresholding*. This approach has been proven to converge for certain problem formulations to a unique minimizer, and this theory serves as a foundation basis for further developments.

For many years the notion of sparsity been central in the motivation and effectiveness of wavelet transforms for compression, denoising, etc. Recently, important results regarding sparsity and L_1-norm minimization have been discovered and are fueling ongoing research activities. A recent issue of the *IEEE Signal Processing Magazine* is dedicated to Compressed Sensing [16], a sparsity-based approach to reduce the number of required measurements of signal.

Some of these discrete-time signal processing applications of wavelet transforms are further discussed in Section 4 of the paper.

As a multiresolution signal analysis technique, the wavelet transform offers the possibility of selective noise filtering and reliable parameter estimation, and therefore, can contribute efficiently to morphological analysis. For this reason wavelets have been extensively used in biomedical signal processing, mainly due to the versatility of the wavelet transform tools. Signal analysis methods derived from wavelet analysis carry large potential to support a wide range of biomedical signal processing applications including noise reduction, feature recognition and signal compression.

In implantable medical devices, such as pacemakers and implantable cardio defibrillators, power consumption is a critical issue due to the limited energy density and the longevity of currently available portable batteries. This implies that the design of such devices has to be optimized for very low-power dissipation. Due to the great relative power required for the analog-to-digital conversion and its marginal improvement in power efficiency over the years, it is predicted that the implementation of a fully digital wavelet signal processor in implantable pacemakers will not be feasible for several decades to come. For this reason, a method for implementing wavelet transform using continuous-time analog circuitry was proposed in Ref. [17], based on the development of ultra-low-power analog integrated circuits that implement the required signal processing, taking into account the limitations imposed by an implantable device.

The implementation of wavelet transform in an analog fashion is equivalent to the implementation of a filter whose impulse response is the desired (reversed) wavelet function. Hence, wavelet transform can be implemented by means of analog filters and filter banks. In implantable medical devices, minimization of the power consumption for a guaranteed performance reduces down to four important design steps as follows,

1. minimization of the total number of wavelet scales required, e.g., by introducing so-called multi-wavelets [18];
2. minimization of the order of the wavelet filter, i.e., finding a suitable approximation to the desired wavelet by means of a low-order rational transfer function [19];
3. optimization of the wavelet filter topology, i.e., finding the optimal state–space description implementing the wavelet filter transfer function [17,20]; and
4. optimizing the elementary wavelet filter building blocks, viz. the integrators.

Other applications that benefit from implementation of wavelet transform by means of analog circuitry are those that deal with high frequencies as required in portable and wearable wireless communication devices, as here the