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Densities in large permutations and parameter testing

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ABSTRACT

A classical theorem of Erdős, Lovász and Spencer asserts that the densities of connected subgraphs in large graphs are independent. We prove an analogue of this theorem for permutations and we then apply the methods used in the proof to give an example of a finitely approximable permutation parameter that is not finitely forcible. The latter answers a question posed by two of the authors and Moreira and Sampaio.
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1. Introduction

Computer science applications that involve large networks form one of the main motivations to develop methods for the analysis of large graphs. The theory of graph limits, which emerged in a series of papers by Borgs, Chayes, Lovász, Sós, Szegedy and Vesztergombi [4,6,5,18], gives analytic tools to cope with problems related to large graphs. It also provides an analytic view of many standard concepts, e.g. the regularity method [19] or property testing algorithms [14,20]. In this paper, we focus

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on another type of discrete objects, permutations, and we give permutation counterparts of some of classical results on large graphs. It is worth noting that not all results on large graphs have permutation analogues and vice versa as demonstrated, for example, by the finite forcibility of graphons and permutons [9] (vaguely speaking, finite forcibility means that a global structure is determined by finitely many substructure densities).

Both our main results are related to the dependence of possible densities of (small) substructures. In the case of graphs, Erdős, Lovász and Spencer [8] considered three notions of substructure densities: the subgraph density, the induced subgraph density and the homomorphism density. They showed that these types of densities in a large graph are strongly related and that the densities of connected graphs are independent in the sense that none of the densities can be expressed as a function of the others. The result has a natural formulation in the language of graph limits, which are called graphons: the body of possible densities of any k connected graphs in graphons, which is a subset of $[0, 1]^k$, has a non-empty interior (in particular, it is full dimensional).

Our first result asserts that the analogous statement is also true for permutations. As in the case of graphs, it is natural to cast our result in terms of permutation limits, called permutons. The theory of permutation limits was initiated in [12,15] (also see [21]) and successfully applied e.g. in [14,17]. To state our first result, we use the notion of a *indecomposable* permutation, which is an analogue of graph connectivity in the sense that an indecomposable permutation cannot be split into *independent parts*. Let T^q be the body of possible densities of indecomposable permutations of order at most q in a permuton (a precise definition and further details can be found in Section 2.1). Our first result says that T^q has a non-empty interior for every q. In particular, it contains $B(\mathbf{w}, \varepsilon)$, for some vector $\mathbf{w}$ and some $\varepsilon > 0$, where $B(\mathbf{w}, \varepsilon)$ denotes the ball of radius ε around $\mathbf{w}$.

Theorem 1. *For every integer $q \geq 2$, there exist a vector $\mathbf{w} \in T^q$ and $\varepsilon > 0$ such that $B(\mathbf{w}, \varepsilon) \subseteq T^q$.*

Our second result is related to algorithms for large permutations. Such algorithms are counterparts of extensively studied graph property testing, see e.g. [2,3,10,11,22]. In the case of permutations, two of the authors and Moreira and Sampaio [13,14] established that every hereditary permutation property is testable with respect to the rectangular distance and two of the other authors [16] strengthened the result to testing with respect to Kendall's tau distance. In addition to property testing, a related notion of parameter testing was also considered in [14] where testable bounded permutation parameters were characterized.

However, the interplay between testing and the finite forcibility of permutation parameters was not fully understood in [14]. In particular, the authors asked [14, Question 5.5] whether there exists a testable bounded permutation parameter that is not finitely forcible. Our second result gives a positive answer to this question.

Theorem 2. *There exists a bounded permutation parameter f that is finitely approximable but not finitely forcible.*

Informally speaking, we utilize the methods used in the proof of Theorem 1 to construct a permutation parameter that oscillates on indecomposable permutations, with bounded amplitude, so that the parameter is testable though it fails to be finitely forcible.

2. Preliminaries

In this section, we introduce the notions used throughout the paper. Most of our notions are standard but we include all of them for the convenience of the reader.

2.1. Permutations

A *permutation of order n* is a bijective mapping from $[n]$ to $[n]$, where $[n]$ denotes the set $\{1, \ldots, n\}$. The order of a permutation σ is denoted by $|\sigma|$. We say a permutation is *non-trivial* if it has order greater than 1. We denote by S_n the set of all permutations of order n and let $\mathfrak{S} = \bigcup_{n\in\mathbb{N}} S_n$. An *inversion* of a permutation σ is a pair (i, j), $i, j \in [|\sigma|]$, such that $i < j$ and $\sigma(i) > \sigma(j)$. An *interval* I in $[m]$ is a

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