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Tensegrity frameworks in one-dimensional space

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ABSTRACT

The edge set of a graph G is partitioned into two subsets $E_C \cup E_S$. A tensegrity framework with underlying graph G and with cables for E_C and struts for E_S is proved to be rigidly embeddable into a one-dimensional line if and only if G is 2-edge-connected and every 2-vertex-connected component of G intersects both E_C and E_S . Polynomial algorithms are given for finding an embedding of such graphs and for checking the rigidity of a given one-dimensional embedding.

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1. Introduction

Tensegrity structures are pin-connected frameworks where some of the members are cables or struts. Today, tensegrity structures interest researchers in the engineering, mathematical and biological communities.

The elements of tensegrity structures, namely cables and struts, are characterized by their abilities to sustain only one type of load, while being capable of deforming freely in the opposite direction. Comparing against the regular pin-connected rod structures, the first property does not present much disadvantage, as in most cases the structures are designed so that the allowed loads induce only one type of force in each of the rods. On the other hand, the second property makes it possible to alter the geometry of the structures and thus to achieve unique technological properties. Controlling the geometry of the static structures gives rise to a variety of practical applications including producing foldable and deployable structures [13], smart structures, structures adjustable to the environmental conditions [11] and many others. Additional advantages of the tensegrity structures include significant weight reduction while not affecting the static performance and simplification of the construction process. Over the past decades, numerous studies of the advantages and the properties were performed, some of which are as follows.

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In engineering, tensegrity structures provide efficient solutions for applications like those in deployable structures [13,6], shape-controllable structures, smart sensors [12] and lightweight structures.

The biological community employs tensegrity structures as models underlying the behavior of a number of biological entities, such as the cytoskeleton [7]. Adopting such models enables biologists to interpret some observed but previously unexplained natural phenomena.

The complexity of the behavior on one hand and the special properties on the other provide the incentive for mathematical studies of tensegrity structures [2,3]. The main interest in this respect is concentrated on the issues of checking rigidity [10,5] and structural analysis of these structures.

A key problem in the design of tensegrity structures is the determination of geometrical configurations where a given structure becomes rigid. This problem, also referred to as the ‘form-finding problem’ [14], does not possess a general analytical solution, except for some special, relatively simple cases [4].

The present paper addresses a combinatorial approach for treating one-dimensional tensegrity structures, i.e. structures where all members are parallel. The paper establishes a theorem for checking the topological rigidity of these structures, i.e. deciding whether for a given graph there exists at least one rigid geometrical embedding. If there does, a graph-theoretical algorithm is provided for finding a rigid embedding for the given frame topology. This can be regarded as an alternative solution for the ‘form-finding problem’, although, for now, it is limited to one-dimensional structures. Additionally, an algorithm for checking the rigidity of a structure with a given geometry is shown to be equivalent to checking whether the corresponding graph is strongly connected.

For any rigid graph there is no one-dimensional singular embedding configuration since for the latter we need that the sum of the virtual work is equal to zero, i.e., the displacement/velocity of a joint is perpendicular to the corresponding rod, a situation that cannot arise in one-dimensional systems.

It is shown that the methodology can partly be considered as a special case of a more general theorem based on matroid theory [10], which raises the possibility that in the future the method could be expanded to multidimensional cases.

2. The condition for graph embeddability as a rigid one-dimensional framework

Let $G = (V, E)$ be a finite graph with vertex set V and edge set E and let χ denote a bipartition $E = E_C \cup E_S$. A function $f : V(G) \rightarrow \mathbb{R}$ is called a *one-dimensional embedding* of G if $x \neq y$ implies $f(x) \neq f(y)$.

A function $g : V(G) \rightarrow \mathbb{R}$ satisfying

$$|g(x) - g(y)| \begin{cases} \leq |f(x) - f(y)| & \text{if } \{x, y\} \in E_C \\ \geq |f(x) - f(y)| & \text{if } \{x, y\} \in E_S, \end{cases} \quad (1)$$

and

$$\text{sign}[g(x) - g(y)] = \text{sign}[f(x) - f(y)] \quad \forall \{x, y\} \in E \quad (2)$$

is called a *motion with respect to the bipartition χ* or for short a χ -*motion* of the embedded graph G . Such a χ -motion is *trivial* if there exists a constant $c \in \mathbb{R}$ such that $g(x) = f(x) + c$ for every $x \in V(G)$.

In the terminology of the real one-dimensional tensegrity structures, the vertices of a graph represent the junctions, while edges belonging to E_C and E_S correspond to cables/struts respectively. The embedding function $f(x)$ indicates the location coordinate of junction x , while the motion function $g(x)$ indicates the new location coordinate of junction x , after the tensegrity structure has been deformed. The requirements of Eqs. (1) and (2) are interpreted as physical constraints for the distance between end junctions of the cables and struts to become only smaller and only larger respectively, while the relative location of the two junctions remains unaltered.

A one-dimensional embedding f is called a *one-dimensional rigid embedding* of G with respect to this bipartition, or for short a *one-dimensional rigid χ -embedding*, if every χ -motion of it is trivial.

A circuit C of the graph G is a *mixed circuit* with respect to a bipartition χ , or for short a χ -*mixed circuit*, if neither $C \cap E_C$ nor $C \cap E_S$ is empty.

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