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Curtis–Tits groups of simply-laced type

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The classification of Curtis–Tits amalgams with connected, triangle free, simply-laced diagram over a field of size at least 4 was completed in [3]. Orientable amalgams are those arising from applying the Curtis–Tits theorem to groups of Kac–Moody type, and indeed, their universal completions are central extensions of those groups of Kac–Moody type. The paper [2] exhibits concrete (matrix) groups as completions for all Curtis–Tits amalgams with diagram $\widetilde{A}_{n-1}$. For non-orientable amalgams these groups are symmetry groups of certain unitary forms over a ring of skew Laurent polynomials. In the present paper we generalize this to all amalgams arising from the classification above and, under some additional conditions, exhibit their universal completions as central extensions of twisted groups of Kac–Moody type.

1. Introduction

A celebrated theorem of Curtis and Tits [8,22] (later extended by Timmesfeld (see [18–21] for spherical groups), by P. Abramenko and B. Mühlherr [1] and Caprace [4] to 2-spherical groups of Kac–Moody type) on groups with finite BN-pair states that these

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groups are central quotients of the universal completion of the concrete amalgam of the Levi components of the parabolic subgroups with respect to a given (twin-) BN-pair. Following Tits [25] (see also [24]) a *group of Kac–Moody type* is by definition a group with RGD system such that a central quotient is the subgroup of $\mathrm{Aut}(\Delta)$ generated by the root groups of an apartment in a Moufang twin-building Δ. This central quotient will be called the associated *adjoint* group of Kac–Moody type.

1.1. Classification of Curtis–Tits amalgams

In [3], for a given connected diagram, we consider all possible rank-2 amalgams which are locally isomorphic to an amalgam arising from the Curtis–Tits theorem in the simply-laced case. Then, extending the techniques from [2], we classify Curtis–Tits amalgams as defined in Definition 2.6 (called Curtis–Tits structures in [2]) with property (D) (see Definition 2.7) and connected simply-laced diagram. We show that any Curtis–Tits amalgam with connected simply-laced diagram without triangles that has a non-trivial universal completion must have property (D). Note that property (D) has no meaning if $|\mathsf{k}| \leq 3$ as all tori equal the center of $\mathrm{SL}_2(\mathsf{k})$. Although many of the results in the present and preceding papers can be proved under the more general assumption of property (D), some rely on 3-sphericity, for instance the simple connectedness of Δ^θ in Section 4 and Lemma 2.10. Note that in the simply-laced case, 3-sphericity is the same as being triangle-free. Indeed, the 3-sphericity is needed in the proof of simple-connectedness of Δ^θ. As noted by one referee it would be desirable to strengthen Lemma 2.10 by removing the triangle-free condition thus yielding the computational tool to compute centers of all simply-laced groups of Kac–Moody type. However, as the study in [2] shows, we cannot exclude the possibility that the universal completion of the universal orientable Curtis–Tits amalgam of type $\widetilde{A}_2$ is a proper central extension of $\mathrm{SL}_3(\mathsf{k}[t, t^{-1}])$.

Assumption 1. Throughout the paper we will make the following assumptions:

1. k is a field with at least 4 elements.
2. $\Gamma = (I, E)$ is a connected simply-laced Dynkin diagram of rank $n = |I|$ without triangles.
3. All Curtis–Tits amalgams have property (D).

In this context, we quote Theorem 1 from [3]:

Classification Theorem. *Let Γ be a connected simply laced Dynkin diagram without triangles and k a field with at least 4 elements. There is a natural bijection between isomorphism classes of universal Curtis–Tits amalgams with property (D) over the field k on the graph Γ and group homomorphisms $\omega\colon \pi(\Gamma, 0) \to C_2 \times \mathrm{Aut}(\mathsf{k})$.*

We'd like to point out that the classification theorem generalizes similar classification results of sound Moufang foundations in [14,25].