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A p-adic interpretation of some integral identities for Hall–Littlewood polynomials ☆

Vidya Venkateswaran

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, United States

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ABSTRACT

If one restricts an irreducible representation V_λ of GL_{2n} to the orthogonal group (respectively the symplectic group), the trivial representation appears with multiplicity one if and only if all parts of λ are even (resp. the conjugate partition λ' is even). One can rephrase this statement as an integral identity involving Schur functions, the corresponding characters. Rains and Vazirani considered q, t-generalizations of such integral identities, and proved them using affine Hecke algebra techniques. In a recent paper, we investigated the $q = 0$ limit (Hall–Littlewood), and provided direct combinatorial arguments for these identities; this approach led to various generalizations and a finite-dimensional analog of a recent summation identity of Warnaar. In this paper, we reformulate some of these results using p-adic representation theory; this parallels the representation-theoretic interpretation in the Schur case. The nonzero values of the identities are interpreted as certain p-adic measure counts. This approach provides a p-adic interpretation of these identities (and a new identity), as well as independent proofs. As an application, we obtain a new Littlewood summation identity that generalizes a classical result due to Littlewood and Macdonald. Finally, our p-adic method also leads to a generalized integral identity

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E-mail address: vidyav@math.mit.edu.

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in terms of Littlewood–Richardson coefficients and Hall polynomials.

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1. Introduction

A crucial problem in representation theory can be described in the following way: let G and H be complex algebraic groups, with an embedding $H \hookrightarrow G$. Also let V be an irreducible representation of G, and W an irreducible representation of H. What information can one obtain about $[V, W] := \dim \mathrm{Hom}_H(W, V)$, the multiplicity of W in V? Here V is viewed as a representation of H by restriction. Such branching rules have important connections to physics as well as other areas of mathematics. There are often beautiful combinatorial objects describing these multiplicities. One prototypical example is that of the symmetric groups $G = S_n$ and $H = S_{n-1}$: the resulting rule has a particularly nice description in terms of Young tableaux.

Two particularly interesting examples involving matrix groups are the restriction of $\mathrm{GL}(2n)$ to $\mathrm{Sp}(2n)$ (the symplectic group) and $\mathrm{GL}(n)$ to $\mathrm{O}(n)$ (the orthogonal group); the combinatorics of these branching rules was first developed by D. Littlewood and continues to be a well-studied and active area at the forefront of algebraic combinatorics and invariant theory. The multiplicities in these branching rules are given in terms of Littlewood–Richardson coefficients, which are combinatorial quantities involving tableaux and lattice permutations. Since the Schur functions s_λ are characters of irreducible polynomial representations of $\mathrm{GL}(2n)$, one may rephrase these rules in terms of Schur functions and symplectic characters (respectively, orthogonal characters). For example, the following theorem computes the multiplicity of the trivial character in the restrictions for these two pairs:

Theorem 1.1. *(See [3,10].)* (1) *For any integer $n \geq 0$, we have*

$$\int_{S \in Sp(2n)} s_\lambda(S) dS = \begin{cases} 1, & \text{if all parts of } \lambda \text{ have even multiplicity} \\ 0, & \text{otherwise} \end{cases}$$

where the integral is with respect to Haar measure on the symplectic group.

(2) *For any integer $n \geq 0$ and partition λ with at most n parts, we have*

$$\int_{O \in O(n)} s_\lambda(O) dO = \begin{cases} 1, & \text{if all parts of } \lambda \text{ are even} \\ 0, & \text{otherwise} \end{cases}$$

(*where the integral is with respect to Haar measure on the orthogonal group*).

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