



ELSEVIER

Contents lists available at ScienceDirect

Journal of Combinatorial Theory, Series A

www.elsevier.com/locate/jcta



Eigenvectors of random matrices: A survey



Sean O'Rourke^a, Van Vu^{b,1}, Ke Wang^{c,d}

^a Department of Mathematics, University of Colorado at Boulder, Boulder, CO 80309, USA

^b Department of Mathematics, Yale University, New Haven, CT 06520, USA

^c Jockey Club Institute for Advanced Study, Hong Kong University of Science and Technology, Hong Kong, China

^d Computing & Mathematical Sciences, California Institute of Technology, Pasadena, CA 91125, USA

ARTICLE INFO

Article history:

Available online 15 July 2016

Keywords:

Eigenvectors
Random matrix
Random graph
Adjacency matrix
Random regular graph

ABSTRACT

Eigenvectors of large matrices (and graphs) play an essential role in combinatorics and theoretical computer science. The goal of this survey is to provide an up-to-date account on properties of eigenvectors when the matrix (or graph) is random.

Published by Elsevier Inc.

Contents

1. Introduction	362
1.1. Overview and outline	365
1.2. Notation	366
2. A toy case: The Gaussian orthogonal ensemble	367
3. Direct comparison theorems	372
4. Extremal coordinates	374
4.1. The largest coordinate	374
4.2. The smallest coordinate	377
5. No-gaps delocalization	377

E-mail addresses: sean.d.ourourke@colorado.edu (S. O'Rourke), van.vu@yale.edu (V. Vu), kewang@ust.hk (K. Wang).

¹ V. Vu is supported by research grants DMS-0901216 and AFOSAR-FA-9550-09-1-0167.

<http://dx.doi.org/10.1016/j.jcta.2016.06.008>

0097-3165/Published by Elsevier Inc.

6.	Random symmetric matrices with non-zero mean	379
6.1.	The largest eigenvector	380
6.2.	Extremal coordinates	380
6.3.	No-gaps delocalization	381
7.	Localized eigenvectors: Heavy-tailed and band random matrices	384
7.1.	Heavy-tailed random matrices	384
7.2.	Random band matrices	385
8.	Singular vectors and eigenvectors of non-Hermitian matrices	386
9.	Random regular graphs	387
10.	Proofs for the Gaussian orthogonal ensemble	388
11.	Tools required for the remaining proofs	396
11.1.	Tools from linear algebra	396
11.2.	Spectral norm	398
11.3.	Local semicircle law	398
11.4.	Smallest singular value	399
11.5.	Projection lemma	399
11.6.	Deterministic tools and the equation $Ax = By$	401
12.	Proofs of results concerning extremal coordinates	401
12.1.	Proof of Corollary 4.4	402
12.2.	Proof of Theorem 4.7	402
13.	Proofs of no-gaps delocalization results	407
13.1.	Proof for Theorem 5.1	407
13.2.	Proof of Corollary 5.4	412
14.	Proofs for random matrices with non-zero mean	418
14.1.	Proof of Theorem 6.4	418
14.2.	Proof of Theorem 6.6	424
14.3.	Proof of Theorem 6.8	430
14.4.	Proof of Theorem 6.10	434
	Acknowledgments	435
	Appendix A. Proof of Lemma 10.3	435
	Appendix B. Proof of Lemmas 11.10 and 11.11	436
	References	439

1. Introduction

Eigenvectors of large matrices (and graphs) play an essential role in combinatorics and theoretical computer science. For instance, many properties of a graph can be deduced or estimated from its eigenvectors. In recent years, many algorithms have been developed which take advantage of this relationship to study various problems including spectral clustering [68,85], spectral partitioning [50,60], PageRank [57], and community detection [52,53].

The goal of this survey is to study basic properties of eigenvectors when the matrix (or graph) is random. As this survey is written with combinatorics/theoretical computer science readers in mind, we try to formalize the results in forms which are closest to their interest and give references for further extensions. Some of the results presented in this paper are new with proofs included, while many others have appeared in very recent papers.

We focus on the following models of random matrices.

Download English Version:

<https://daneshyari.com/en/article/4655054>

Download Persian Version:

<https://daneshyari.com/article/4655054>

[Daneshyari.com](https://daneshyari.com)