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Topological spaces associated to higher-rank graphs [☆]

Alex Kumjian [a], David Pask [b], Aidan Sims [b], Michael F. Whittaker [b,1]

[a] *Department of Mathematics (084), University of Nevada, Reno NV 89557-0084, USA*
[b] *School of Mathematics and Applied Statistics, The University of Wollongong, NSW 2522 Australia*

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ABSTRACT

We investigate which topological spaces can be constructed as topological realisations of higher-rank graphs. We describe equivalence relations on higher-rank graphs for which the quotient is again a higher-rank graph, and show that identifying isomorphic co-hereditary subgraphs in a disjoint union of two rank-k graphs gives rise to pullbacks of the associated C^*-algebras. We describe a combinatorial version of the connected-sum operation and apply it to the rank-2-graph realisations of the four basic surfaces to deduce that every compact 2-manifold is the topological realisation of a rank-2 graph. We also show how to construct k-spheres and wedges of k-spheres as topological realisations of rank-k graphs.
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E-mail addresses: alex@unr.edu (A. Kumjian), dpask@uow.edu.au (D. Pask), asims@uow.edu.au (A. Sims), Mike.Whittaker@glasgow.ac.uk (M.F. Whittaker).

[1] Current address: School of Mathematics and Statistics, University of Glasgow, 15 University Gardens, Glasgow Q12 8QW, United Kingdom.

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1. Introduction

Higher-rank graphs, also called k-graphs, were introduced by Kumjian and Pask [7] as combinatorial models for higher-rank Cuntz–Krieger algebras. Since then, the resulting class of C^*-algebras has been studied in detail. More recently, in [13,14,6,8], an investigation of k-graphs from a topological point of view was begun. Definition 3.2 of [6] associates to each k-graph Λ a topological realisation X_Λ whose fundamental group and homology are the same as the fundamental group and cubical homology of Λ. The motivation for the current article was to investigate the range of topological spaces which can be constructed as topological realisations of k-graphs. We began with two goals: obtain all compact 2-manifolds as topological realisations of 2-graphs; and, more generally, obtain all triangularisable k-manifolds as topological realisations of k-graphs.

Our approach to the first goal was to exploit the classification of compact 2-manifolds as spheres, n-holed tori, or connected-sums of the latter with the Klein bottle or projective plane (see for example [10, Theorem I.7.2]). Examples of 2-graphs whose topological realisations are homeomorphic to each of the four basic surfaces were presented in [6]. So our aim was to develop a combinatorial connected-sum operation for 2-graphs, and to show that it can be applied to finite disjoint unions of the four 2-graphs just mentioned so as to construct any desired connected sum of their topological realisations. We achieve this in Section 4.

Our approach to the second, more general, goal consisted of four steps. Step 1 was to determine which equivalence relations on k-graphs have the property that the quotient category itself forms a k-graph. Step 2 was to invoke [6, Proposition 5.3]—which shows that topological realisation is a functor from k-graphs to topological spaces—to see that, for k-graphs, topological realisations of quotients coincide with quotients of topological realisations. Step 3 was to construct k-graphs Σ_k whose topological realisations are naturally homeomorphic to k-simplices. Step 4 was to show that equivalence relations corresponding to desired identifications amongst the $(k-1)$-faces of a disjoint union of copies of Σ_k are of the sort developed in Step 1; and then deduce that arbitrary triangularisable manifolds could be realised as the topological realisations of appropriate quotients of disjoint unions of copies of Σ_k. We have achieved steps 1–3, but the combinatorics of k-graphs place significant constraints on the ways in which faces in a disjoint union of copies of the k-graphs Σ_k from Step 3 can be identified so as to produce a new k-graph, so we are as yet unable to realise arbitrary triangularisable manifolds. However, our construction is flexible enough so that we can glue two k-simplices on their boundaries to obtain a k-sphere. The details of this appear in section 5.

The paper is organised as follows. In Section 2 we identify those equivalence relations $\sim$ on a k-graph Λ for which the quotient $\Lambda/{\sim}$ forms a k-graph. Proposition 2.3 shows that the quotient operation is well-behaved with respect to the topological realisation of a k-graph. In Section 3 we investigate the properties of quotients of k-graphs at the level of their associated C^*-algebras. Specifically, given k-graphs Λ_1 and Λ_2 with a partially-defined isomorphism ϕ between the complements in the Λ_i of hereditary subsets

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