Journal of Combinatorial Theory, Series A 135 (2015) 237–267

Contents lists available at ScienceDirect

Journal of Combinatorial Theory, Series A

www.elsevier.com/locate/jcta

Multi-wise and constrained fully weighted Davenport constants and interactions with coding theory ☆

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A R T I C L E I N F O

Article history:
Received 7 July 2014
Available online 11 June 2015

Keywords:
Finite abelian group
Weighted subsum
Zero-sum problem
Davenport constant
Linear intersecting code
Cap set

A B S T R A C T

We consider two families of weighted zero-sum constants for finite abelian groups. For a finite abelian group $(G, +)$, a set of weights $W \subset \mathbb{Z}$, and an integral parameter m, the m-wise Davenport constant with weights W is the smallest integer n such that each sequence over G of length n has at least m disjoint zero-subsums with weights W. And, for an integral parameter d, the d-constrained Davenport constant with weights W is the smallest n such that each sequence over G of length n has a zero-subsum with weights W of size at most d. First, we establish a link between these two types of constants and prove several basic and general results on them. Then, for elementary p-groups, establishing a link between our constants and the parameters of linear codes as well as the

☆ The research of O. Ordaz is supported by the Postgrado de la Facultad de Ciencias de la U.C.V., project No. PFC-03-05, and the Banco Central de Venezuela; the one of W.A. Schmid, by the ANR project Caesar, project number ANR-12-BS01-0011.

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http://dx.doi.org/10.1016/j.jcta.2015.05.004

cardinality of cap sets in certain projective spaces, we obtain various explicit results on the values of these constants.

1. Introduction

For a finite abelian group $(G, +)$ the Davenport constant of G is the smallest n such that each sequence $g_1 \dots g_n$ over G has a non-empty subsequence the sum of whose terms is 0. This is a classical example of a zero-sum constant over a finite abelian group, and numerous related invariants have been studied in the literature. We refer to [16] for a survey of the subject.

An example is the multi-wise Davenport constants: the m-wise Davenport constant, for some integral parameter m, is defined like the Davenport constant yet instead of asking for *one* non-empty subsequence with sum 0 one asks for m disjoint non-empty subsequences with sum 0. These constants were first considered by Halter-Koch [23] due to their relevance in a quantitative problem of non-unique factorization theory. Delorme, Quiroz and the second author [11] highlighted the relevance of these constants when using the inductive method to determine the Davenport constant itself.

Other variants are constants defined in the same way as the Davenport constant, yet imposing a constraint on the length of the subsequence whose sum is 0. Classical examples are generalizations of the well-known Erdős–Ginzburg–Ziv theorem, where one seeks zero-sum subsequences of lengths equal to the exponent of the group or also equal to the order of the group. Another common constraint is to impose an upper bound on the length, for example again the exponent of the group, yielding the η-invariant of the group. Here, we refer to these constants as constrained Davenport constants, more specifically the d-constrained Davenport constant is the constant that arises when one asks for the existence of a non-empty zero-sum subsequence of length at most d. There are numerous contributions to this problem and we refer to [16, Section 6] for an overview.

In addition to these classical zero-sum constants, in recent years there was considerable interest in weighted versions of these constants. There are several ways to introduce weights in such problems. One that received a lot of interest lately is due to Adhikari et al. (see [1,2,31,32] for some contributions, and [33] for a more general notion of weights) where for a given set of weights $W \subset \mathbb{Z}$ one asks for the smallest n such that a sequence $g_1 \dots g_n$ over G has a W-weighted subsum that equals 0, that is there exists a subsequence $g_{i_1} \dots g_{i_k}$ and $w_j \in W$ such that $\sum_{j=1}^{k} w_j g_{i_j} = 0$, yielding the W-weighted Davenport constant of G. And, analogously, one defines the m-wise W-weighted Davenport constant and the d-constrained W-weighted Davenport (see Definition 3.1 for a more formal definition). We refer to [22] for an overview on weighted zero-sum problems, including a more general notion of weights, and to [24] for an arithmetical application of a weighted Davenport constant.