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Immersion in four-edge-connected graphs

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Fix $g > 1$. Every graph of large enough tree-width contains a $g \times g$ grid as a minor; but here we prove that every four-edge-connected graph of large enough tree-width contains a $g \times g$ grid as an immersion (and hence contains any fixed graph with maximum degree at most four as an immersion). This result has a number of applications.

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1. Introduction

Let G, H be graphs. (All graphs in this paper are finite, possibly with loops or parallel edges.) A *weak immersion* of H in G is a map η, with domain $V(H) \cup E(H)$, mapping

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each vertex of H to a vertex of G, and each edge of H to a path or cycle of G, satisfying the following:

- $\eta(u) \neq \eta(v)$ for all distinct $u, v \in V(H)$;
- for each $e \in E(H)$ with distinct ends u and v, $\eta(e)$ is a path of G with ends $\eta(u)$, $\eta(v)$;
- for each loop in H with end v, $\eta(e)$ is a cycle of G passing through $\eta(v)$; and
- for all distinct $e, f \in E(H)$, $E(\eta(e) \cap \eta(f)) = \emptyset$.

If in addition we have

- for all $v \in V(H)$ and $e \in E(H)$, if e is not incident with v in H then $\eta(v) \notin V(\eta(e))$

then η is called a *strong immersion*. This paper is only concerned with strong immersion, and from now on we omit "strong", and just speak of "immersion". If there is an immersion of H in G, we say that "H can be immersed in G" and "G contains H as an immersion" (or just "G immerses H"). If in addition, for all distinct $e, f \in E(H)$, every vertex of $\eta(e) \cap \eta(f)$ is equal to $\eta(v)$ for some $v \in V(H)$ incident in H with both e and f, then η is called a *subdivision map* of H in G.

If $g > 1$ is an integer, the $g \times g$ *grid* is a graph with vertex set $\{v_{ij} \ : 1 \leq i, j \leq g\}$, where v_{ij} is adjacent to $v_{i'j'}$ if $|i - i'| + |j - j'| = 1$. We denote this graph by J_g.

A *tree-decomposition* of a graph G is a pair $(T, (W_t \ : t \in V(T)))$, such that

- T is a tree
- $W_t \subseteq V(G)$ for each $t \in V(T)$
- $V(G) = \bigcup(W_t \ : t \in V(T))$
- for every edge uv of G, there exists $t \in V(T)$ with $u, v \in W_t$
- for $t, t', t'' \in V(T)$, if t' belongs to the path of T between t and t'', then $W_t \cap W_{t''} \subseteq W_{t'}$.

We call $\max(|W_t| - 1 \ : t \in V(T))$ the *width* of the tree-decomposition, and say that G has *tree-width* k if k is minimum such that G admits a tree-decomposition of width k.

We say that H is a *minor* of G if a graph isomorphic to H can be obtained from a subgraph of G by contracting edges. The following is well-known [3]:

1.1. *For all $g > 1$ there exists k such that every graph with tree-width at least k contains J_g as a minor.*

(Note that this is sharp in the sense that for all k there exists g such that no graph of tree-width less than k contains J_g as a minor.) In this paper we prove a similar result for immersion, the following. (Two versions of this result were found independently by two subsets of the authors, and one of these versions appears in [1].)

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