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Are there any good digraph width measures?



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ABSTRACT

Many width measures for directed graphs have been proposed in the last few years in pursuit of generalizing (the notion of) treewidth to directed graphs. However, none of these measures possesses, at the same time, the major properties of treewidth, namely,

1. being *algorithmically useful*, that is, admitting polynomial-time algorithms for a large class of problems on digraphs of bounded width (e.g. the problems definable in MSO_1);
2. having nice *structural properties* such as being (at least nearly) monotone under taking subdigraphs and some form of arc contractions (property closely related to characterizability by particular cops-and-robber games).

We investigate the question whether the search for directed treewidth counterparts has been unsuccessful by accident, or whether it has been doomed to fail from the beginning. Our main result states that any *reasonable* width measure for directed graphs which satisfies the two properties above

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must necessarily be similar to treewidth of the underlying undirected graph.

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1. Introduction

An intensely investigated field in algorithmic graph theory is the design of graph *width parameters* that satisfy two seemingly contradictory requirements: (1) a large class of problems must be efficiently solvable on the graphs of bounded width and (2) graphs of bounded width should have a nice, reasonably rich and natural structure.

For undirected graphs, research into width parameters has been extremely successful with several algorithmically useful measures being proposed over the years, chief among them being treewidth [43], branchwidth [44] and clique-width [11] (see also [6,25]). Many problems that are hard on general graphs turned out to be tractable on graphs of bounded treewidth. These results were generalized by Courcelle’s celebrated theorem which states that a (very) large class of problems, i.e. the class of all MSO_2 -definable problems, is tractable on graphs of bounded treewidth [9]. Treewidth and branchwidth are closely related and their very nice structural properties are well known. Furthermore, the tractability of all MSO_2 problems closely characterizes the class of graphs of bounded treewidth [35].

However, for *directed graphs* no single known *width measure* is as successful as treewidth is for undirected graphs. We feel the reason for this is that all the currently known digraph width measures fail on at least one of the aforementioned conditions (1) and (2).

During the last 15 years, many digraph width measures inspired by treewidth were introduced, the prominent ones being directed treewidth [29], DAG-width [4,40], and Kelly-width [28]. These width measures proved useful for some problems. For instance, one can obtain polynomial-time (XP to be more precise, see Section 2) algorithms for HAMILTONIAN PATH on digraphs of bounded directed treewidth [29] and for PARITY GAMES on digraphs of bounded DAG-width [5] and Kelly-width [28]. But there is the negative side, too. HAMILTONIAN PATH, for instance, probably cannot be solved [37] on digraphs of directed treewidth, DAG-width, or Kelly-width at most k in time $\mathcal{O}(f(k) \cdot n^c)$, where c is a constant independent of k . Note that HAMILTONIAN PATH *can* be solved in such a running time for undirected graphs of treewidth at most k [9].

Moreover, for the former ones and even new measures DAG-depth and Kenny-width¹ [22], which are much more restrictive than DAG-width; (1) problems such as DIRECTED DOMINATING SET, DIRECTED CUT, ORIENTED CHROMATIC NUMBER 4, MAX/MIN LEAF OUTBRANCHING, or k -PATH remain NP-complete on digraphs of con-

¹ Kenny-width of [22] is a different measure than Kelly-width of [28].

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