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Fast track article

Connectivity in time-graphs[☆]

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ABSTRACT

Dynamic networks are characterized by topologies that vary with time and are represented by *time-graphs*. The notion of connectivity in time-graphs is fundamentally different from that in static graphs. End-to-end connectivity is achieved opportunistically by the store–carry-forward paradigm if the network is so sparse that source–destination pairs are usually not connected by complete paths. In static graphs, it is well known that the network connectivity is tied to the spectral gap of the underlying adjacency matrix of the topology: if the gap is large, the network is well connected. In this paper, a similar metric is investigated for time-graphs. To this end, a time-graph is represented by a 3-mode reachability tensor which indicates whether a node is reachable from another node in t steps. To evaluate connectivity, we consider the expected hitting time of a random walk, and the time it takes for epidemic routing to infect all vertices. Observations from an extensive set of simulations show that the correlation between the second singular value of the matrix obtained by unfolding the reachability tensor and these indicators is very significant.

1. Introduction

In wireless mobile networks, end-to-end connectivity is achieved collectively without the need for an established infrastructure using self-configuring applications and protocols (i.e. routing). Because of node mobility and other forms of dynamism in the network topology, the information these protocols use changes frequently. Rather than fetching more recent information at the cost of higher overhead, the protocols may employ opportunistic methods to cope with dynamism [1]. In addition, the density of the network may be low such that source–destination pairs are not connected by complete paths most of the time. In such intermittently connected networks, end-to-end connectivity is achieved *over time* by utilizing the *store–carry-forward* paradigm.

It is useful for many applications to characterize how well the network is connected. For example, in well connected networks, epidemic algorithms quickly spread the messages to the network and the minimum and/or maximum time needed to spread information to the whole network is small; mechanisms that are used to estimate or optimize a parameter converge quickly and the information flow is fast. Similarly, a random walk based mechanism, in which a random walker moves to neighboring nodes with equal probabilities, quickly terminates with success if the network is well connected. In intermittently connected networks, even though there may be no complete paths between source–destination pairs at any given time instant, the messages are delivered relatively quickly to the destinations if the network is well mixed.

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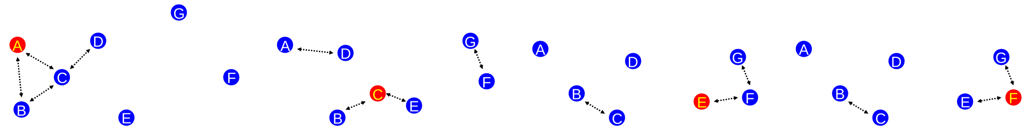

(a) Network topology at t_1. (b) Network topology at t_2. (c) Network topology at t_3. (d) Network topology at t_4.

Fig. 1. Evolution of a network over time. *A* and *F* are never connected. Still, end-to-end connectivity can be maintained between these nodes over time.

In dynamic networks, the network topology constantly evolves, typically in a non-deterministic manner, by inserting or deleting edges and/or nodes over time, and the notion of connectivity is different from static networks. Consider the snapshots taken from a mobile network, that are depicted in Fig. 1. In this example, nodes *A* and *F* are never connected at any time instant. However, node *A* can send a packet to *C* at t_1. At this time, *C* has no neighbor that it can forward this packet for delivery to *F*. *C* keeps the packet until t_2 in its buffer and sends it to *E* at this time. At t_3, *E* transmits the packet to node *F*. This delivery method exploits the dynamism in the network for end-to-end connectivity. This is called store–carry-forward paradigm and is widely used in routing protocols for networks with intermittent connectivity [2]. Even though complete paths between a majority of node pairs at any given instant do not exist in this case, the links between the nodes might be formed so that a message originating at one node is delivered to another node over time.

A dynamic, variable topology network is represented by a *time-graph*, which indicates the creation and deletion of the vertices and/or edges over time. In particular, we use 3-mode adjacency tensors or three-dimensional arrays to model time-graphs and relate their structure to the network connectivity. The tensor indicates whether two nodes are connected by a common link at a given time, similar to the adjacency matrix of a static graph, i.e. the entry $\mathcal{A}_{ijk} = 1$ in the adjacency tensor if vertex *i* is connected to vertex *j* at time *k*.

To provide some intuition behind our work, we start with the motivating scenario of *d*-regular, undirected graphs with *n* vertices and *nd* edges (recall that a *d*-regular graph has *d* edges associated with each vertex). It is well known that the associated graph adjacency matrix has the first eigenvalue exactly equal to *d*. If the difference between this first eigenvalue and the second largest one is sufficiently large, then the graph is an expander. Intuitively, this implies that the number of edges that must be removed from the graph in order to make some large subset of its vertices disconnected from the remainder of the graph is also large. Thus, expanders can represent robust networks with good connectivity properties, while at the same time having only a small number of edges (e.g., linear in *n*). Our work explores whether a similar property holds for evolving graphs, by studying properties of such graphs and their connections to tensor (as opposed to matrix) eigenvalues.

As a general rule, if the network is well connected, an opportunistic method such as random walk performs better. This quality stems from the fact that a random walk is able to sample nodes in a network with respect to a (typically uniform) probability distribution in a small number of steps in well connected networks. Hence, the performance of the random walk indicates how well the network is connected. In well connected graphs, the *hitting time*, i.e. the number of steps a random walk takes before visiting a particular subset of the graph vertices for the first time. In dynamic networks, the forwarding probabilities are derived from the adjacency tensor and continuously change over time. Therefore, a random walk follows a non-homogeneous Markov Chain. A similar opportunistic method is epidemic routing [3], where nodes hold on to the messages they receive and replicate them in the nodes they subsequently meet. In a well connected network, all nodes quickly obtain a replica of the original message. On the other hand, the structural elements of the network are deduced through a series of operations on the adjacency tensor. Using the information given by the adjacency tensor, we can obtain the reachability tensor, which indicates whether a random walk starting from one node can reach another node after *t* time steps. We normalize the rows of the matrices of this tensor, and unfold the tensor around the "distinguished" mode or dimension [4], which in this case is the dimension that depicts time. This operation yields a two-dimensional matrix. We use the singular values of this matrix as the structural metrics of the time-graphs.

Our observations, based on data from extensive simulations, show that the correlation between the second singular value of this matrix and the expected hitting time is very high, above 0.9, which is a very large correlation. Similarly, the average time until the epidemic routing delivers a message that originates at an arbitrary node to the entire network is highly correlated with this structural metric, with a correlation coefficient more than 87%. Hence, the second singular value of the unfolded reachability tensor is a good indicator of network connectivity. Performing these experiments, we used a variety of node densities and speeds. This way, we are able to evaluate this structural metric in scenarios where the nodes are always instantaneously connected via complete paths or they are intermittently connected over time. Our experiments show that the proposed singular value can be used to evaluate the connectivity of dynamic networks. Even though tensors have been drawing a lot of interest recently, researchers still have a long way to go towards understanding the algebraic properties of the tensors. Therefore, it is not possible to support these observations with theoretical proof yet.

The rest of this paper is organized as follows. In the next section we review the related work. In Section 3 we introduce our time-graph model, explain how the expected hitting time on time-graphs is derived and present the notion of reachability tensor. In Section 4 we show that the hitting time is highly correlated to the structural properties of the reachability tensor via data obtained from simulations with various mobility models. Section 5 concludes the paper and discusses the open problems.