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Coloring perfect graphs with no balanced skew-partitions

Maria Chudnovsky [a,1], Nicolas Trotignon [b], Théophile Trunck [c], Kristina Vušković [d,e,2]

[a] *Princeton University, 68 Washington Road, Princeton, NJ 08544, USA*
[b] *CNRS, LIP, ENS Lyon, INRIA, Université de Lyon, France*
[c] *ENS Lyon, LIP, INRIA, Université de Lyon, France*
[d] *School of Computing, University of Leeds, Leeds, UK*
[e] *Faculty of Computer Science (RAF), Union University, Belgrade, Serbia*

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ABSTRACT

We present an $O(n^5)$ algorithm that computes a maximum stable set of any perfect graph with no balanced skew-partition. We present $O(n^7)$ time algorithm that colors them.

E-mail address: nicolas.trotignon@ens-lyon.fr (N. Trotignon).
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1. Introduction

A graph G is *perfect* if every induced subgraph G' of G satisfies $\chi(G') = \omega(G')$. In the 1980s, Gröstchel, Lovász, and Schrijver [10] described a polynomial time algorithm that colors any perfect graph. A graph is *Berge* if none of its induced subgraphs, and none of the induced subgraphs of its complement, is an odd chordless cycle on at least five vertices. Berge [2] conjectured in the 1960s that a graph is Berge if and only if it is perfect. This was proved in 2002 by Chudnovsky, Robertson, Seymour and Thomas [7]. Their proof relies on a *decomposition theorem*: every Berge graph is either in some simple basic class, or has some kind of decomposition. In 2002, Chudnovsky, Cornuéjols, Liu, Seymour and Vušković [6] described a polynomial time algorithm that decides whether any input graph is Berge. The method used in [10] to color perfect graphs (or equivalently by [7], Berge graphs) is based on the ellipsoid method, and so far no purely combinatorial method is known. In particular, it is not known whether the decomposition theorem from [7] may be used to color Berge graphs in polynomial time.

This question contains several potentially easier questions. Since the decomposition theorem has several outcomes, one may wonder separately for each of them whether it is helpful for coloring. The basic graphs are all easily colorable, so the problem is with the decompositions. One of them, namely the balanced skew-partition, seems to be hopeless. The other ones (namely the 2-join, the complement 2-join and the homogeneous pair) seem to be more useful for coloring and we now explain the first step in this direction. Chudnovsky [4,5] proved a decomposition theorem for Berge graphs that is more precise than the theorem from [7]. Based on this theorem, Trotignon [19] proved an even more precise decomposition theorem, that was used by Trotignon and Vušković [20] to devise a polynomial algorithm that colors Berge graphs with no balanced skew-partition, homogeneous pair nor complement 2-join. This algorithm focuses on the 2-join decompositions. Here, we strengthen this result by constructing a polynomial time algorithm that colors Berge graphs with no balanced skew-partition.

Our algorithm is based directly on [4,5], and a few results from [19] and [20] are used. It should be pointed out that the method presented here is significantly simpler and shorter than [20], while proving a more general result. This improvement is mainly due to the use of *trigraphs*, that are graphs where some edges are left "undecided". This notion introduced by Chudnovsky [4,5] helps a lot to handle inductions, especially when several kinds of decompositions appear in an arbitrary order.

It is well known that an $O(n^k)$ algorithm that computes a maximum weighted stable set for a class of perfect graphs closed under complementation, yields an $O(n^{k+2})$ algorithm that computes an optimal coloring. See for instance [13,17] or Section 8 below. This method, due to Gröstchel, Lovász, and Schrijver, is quite effective and combinatorial. Hence, from here on we just focus on an algorithm that computes a maximum weighted stable set. Also, in what follows, in order to keep the paper as readable as possible, we construct an algorithm that computes the weight of a maximum weighted stable set, but does not output a set. However, all our methods are clearly constructive, so our algorithm may easily be turned into an algorithm that actually computes the desired stable set.